JOURNAL OF BALKAN GEOPHYSICAL SOCIETY, Vol.8, No 4, November 2005, p.139-148

Image enhancement in seismic tomography by grid handling: Synthetic simulations with fault-like structures

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Abstract: *In seismology the accurate mapping of faults and fault zones plays an important role as a crucial step for characterizing the earthquake potential of an area. Cross-hole seismic surveys for engineering site investigations can provide high resolution images of subsurface seismic velocity capable of delineating tectonic structures such as vertical or dipping faults juxtaposing different geological formations or step-like structures with good accuracy. The result of velocity reconstruction in travel time seismic tomography is restricted by factors such as ray aperture and distribution. These physical limitations cannot be completely surmounted even in cross borehole acquisitions where the ray coverage is the best one possible. The present paper studies the effects of staggering normal grids as a tool to increase resolution and reduce inversion vagueness and instability. Truncated Singular Value Decomposition as a mathematical tool of least demands on the final solution is utilized, and the inversion scheme is examined with respect to the number of available data. Numerical simulations of a model featuring a fault-like structure are performed and the resulting recovered images are compared against straight grid inversions. Reconstructed images using staggered grids provide a smoothed version of the true model since they basically operate as a moving average filter on the model. On the other hand the final tomograms of the synthetic tests, as a result of shifting the grid, show high accuracy and resolution since both the geometry of the fault and the velocity values throughout the model are better determined. Conventional grid inversion fails to image properly regions of reduced ray coverage, and in general a considerably blurred image is generated. By staggering the grid we can enhance image quality and reduce possible nonuniqueness of solutions without imposing any constraining conditions such as smoothing or damping, retrieving in that way all possible information from our data without the risk to lead to a preferred result.*

Key words: *Forward Seismic Modelling and Inversion, Image Enhancement, Staggered Grids.*

INTRODUCTION

Seismic traveltime inversion has developed into a valuable tool for imaging the spatial distribution of seismic velocities in complex geological environments; yet the severe problem of nonuniqueness is present in tomographic investigations. Factors such as noisy measurements and acquisition accuracy errors can be mostly overcome. On the other hand, uneven distribution of seismic sources and receivers, missing data, and ray bending, for a given model parameterization, remain the most serious problems that will give rise to the null space, leading to restricted resolution and increase of nonuniqueness in the final tomograms.

Overcoming the nonuniqueness in solving ill-posed inverse problems is a central goal for research in geophysical inverse theory. A conventional way of dealing with these problems is to damp or smooth the solution, in which the damping factors and the smoothing operators are usually chosen by trial and error, or based on the relationship between the model variance and the solution variance. Unavoidably, these schemes deform the model resolution and make the inverse procedure biased since they affect not only the zones inadequately constrained by the data, but the whole model. Alternative approaches based on grid manipulation have been proposed to decrease the mismatch between the ray coverage and the utilized parameterization in the tomographic procedure. Among them we cite Abers and Roecker (1991), Bohm et al. (2000), Michelini (1993, 1995), Vesnaver (1996), Sambridge and Gudmundsson (1998), Zhou (2003) Thurber and Eberhart-Phillips (1999), Vesnaver and Böhm (2000).

The most common method to discretize the earth in seismic tomography is the regular constant-slowness cell approach. Choosing the number and shape of model parameters as well as the position of the grid itself is restricted by the number of available data and physical limits of possible model resolution. Shifting the grid gives us an advantage over a more detailed parameterization since the staggered grids provide us a somewhat different recovered image each time we move the grid boundaries, without changing the number of model cells or the area under investigation. On the other hand, a more refined model with a large number of cells and higher resolution can cause inversion ambiguities and instabilities since some cells are not crossed by any rays while many of them are crossed by almost parallel ones, providing in that way linearly depended equations and in general a incorrect image.

In the following, the idea of the staggered grid methodology (Vesnaver and Böhm 2000) is demonstrated with synthetic examples using a cross-hole configuration on a model featuring a normal fault-like structure as a tool to reduce the vagueness and increase resolution in seismic tomography. Normal grid approaches are compared against staggered grids utilizing the truncated singular value decomposition approach.

MOVING THE GRID

During the blocky parameterization of the earth for tomographic purposes the number of cells inside the model should be considered to be as much as the number of available ray paths for a somehow safe inversion result (Vesnaver and Böhm, 2000). In cases of more fine models, certain regions inside the model are poorly sampled by rays, spanning in this way the null space of model parameters. This trade of between parameter number and available data is even more restricted by factors such as the seismic wavelength, acquisition and picking errors. Even thought the real model is a continuous function, local estimates of physical parameters such as velocity and density, averaged in constant velocity pixels, can be estimated.

In Figure 1, we consider the case of a model parameterized in 15 x 20 pixels and a portion of it including 4 pixels.

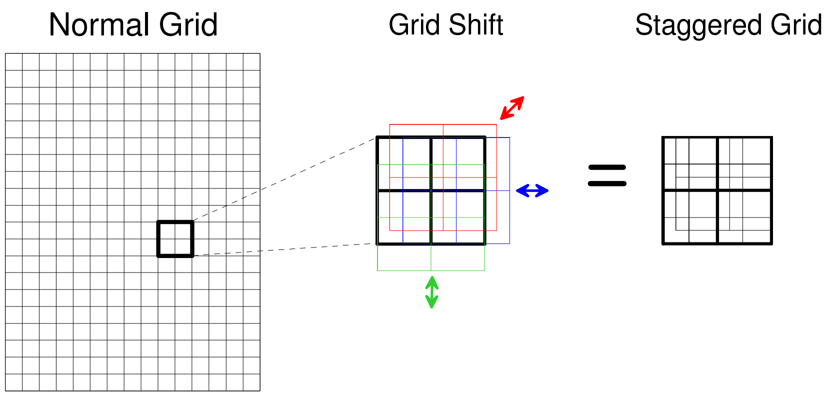

FIG. 1. By staggering pixel boundaries, different images of the same object arise.

Moving the grid in different directions keeps the number of parameters to invert each time the same. The imaged area remains identical while the ray path segments inside each shifted pixel slightly change during each grid movement. The result of these altered ray paths is a somewhat different image reconstruction during the inversion process. Summing up the recovered pictures by staggering the grid we can get a more detailed representation of the model with improved resolution and enhanced geometrical velocity structures. The advantage of this technique is that we improve the final tomogram without imposing any constrains or biasing the solution and in general interfering with the inversion.

A good initial parameterization where the number, shape and position of model parameters ensures an over determined system of normal equations, when followed by staggered grid inversions, can significantly reduce the solution vagueness and non uniqueness, producing in that way better resolved and more reliable velocity models.

FORWARD, INVERSE MODELING AND THE TRUNCATED SINGULAR VALUE DECOMPOSITION METHOD

A shortest path algorithm (SPR), using the graph theory (Moser, 1991) and Fermat's principle, is used to trace ray paths and to approximate the problem into calculating traveltime routes through a given network of nodes. The possible nodes that a ray can pass lie on the sides of each cell. The problem of finding the shortest paths for a given source is solved using Dijkstra algorithm (Dijkstra, 1959). Even through ray tracing using graph theory is a very time consuming step, SPR has the advantages that all shortest paths from one source are constructed simultaneously, a fact that suits best cross-hole experiments.

A least square solution for the system is obtained by the use of the Singular Value Decomposition method, SVD (Golub and Reinch, 1970). The matrix A (Jacobian matrix of partial derivatives) can be factored into a product of three other matrices:

$$A = U \Lambda V^t$$

where the n data $U_{(nxp)}$ and p parameters $V_{(pxp)}$ are respectively the data space and parameter space singular vectors and Λ is a pxp diagonal matrix containing at most $r \leq p$ singular values.

In that case the least squares solution is given by

$$X = V\Lambda^{-1}U^{t}y$$

where y is the vector whose components are the differences between observed and modeled travel times and x is the parameter update vector. Because of the ill-poseness of the system, a certain part of the singular values is practically zero causing instability to the Λ matrix when inverted.

The method of truncated singular value decomposition (SVD) is employed for the tomographic inversion of travel time data. The signal decomposition capability of SVD is exploited to extract the significant feature components by decomposing the Jacobian matrix into a set of basic patterns (singular vectors) with associated scaling factors (singular values). The threshold value for retaining significant information is found by trial and error through the singular value spectra values. As a rule of thumb singular values in the vicinity of an abrupt change in the spectra curve are tested and the recovered velocity tomogram with the smallest RMS data error provides the smallest singular value that contributes to the solution. The remaining values correspond to vectors that are assumed to span the null space of model and data parameters and can be used to identify geological patterns that can not be recovered in the current experiment.

NUMERICAL EXPERIMENTS

Numerical experiments were used to compare the normal and staggered grid solutions under the same initial conditions. Figure 2a shows a 2D synthetic velocity structure representing a normal seismic fault with an abrupt (100%) velocity contrast between the two formations.

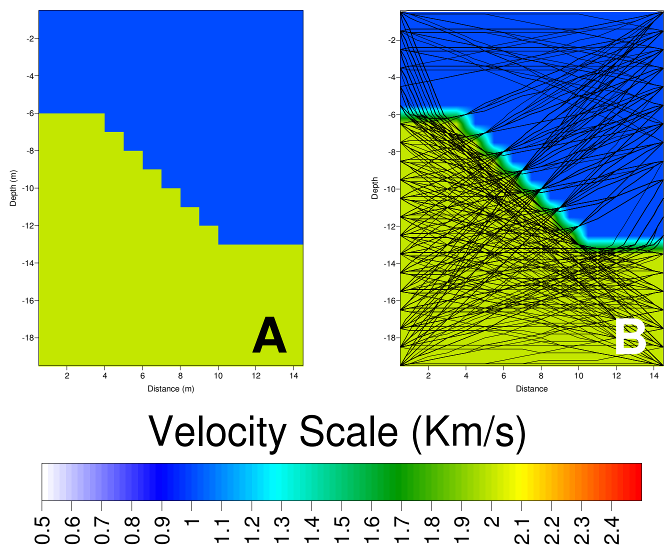

FIG. 2. A 2-D velocity model representing a normal fault structure (left) and the raypaths of a simulated cross-hole survey (right).

The acquisition geometry (Fig. 2b) represents a typical shallow cross-hole seismic survey where 20 sources and 20 receivers are placed on opposite sides of the imaging area at a space increment of 1 meter. Data are contaminated with uniform random noise (~4%), with zero mean and amplitude ±*b*, where *b* is a value chosen based on the ratio of maximum-minimum observed travel-times. The model was discretized in a 15 x 20 grid of constant slowness square pixels 1 x 1m in size giving a total of 300 inversion variables to check for. Ray coverage is good in most regions of the target area (Fig. 2b). First arrival travel times were generated by incorporating SPR algorithm. Figure 3 shows travel time curves for five shots. Distances are given in meters, velocities in km/s and traveltimes in ms.

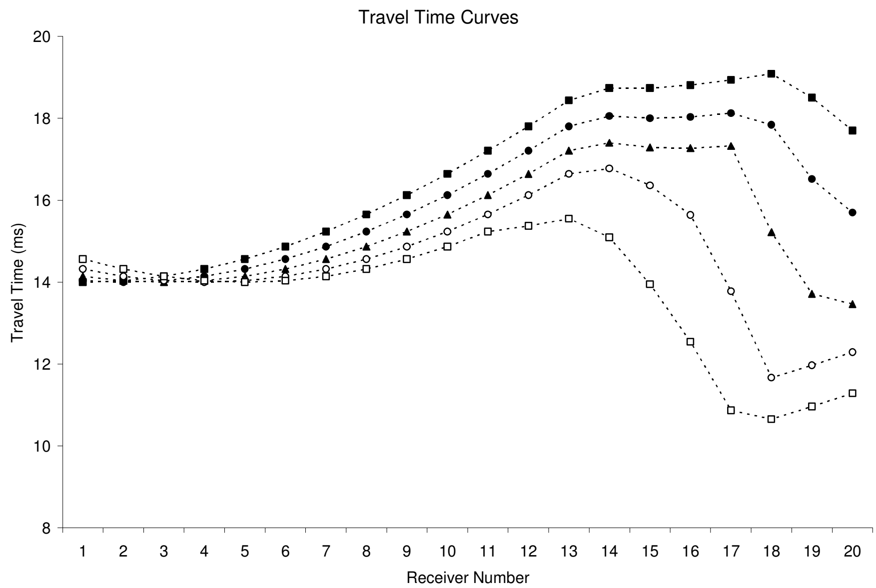

FIG. 3. Travel time curves for five shots of the cross-hole velocity model.

Starting from the same constant velocity (1.5 km/s) reference model and using the same data, both normal and staggered grid inversions were carried out. Figure 4 illustrates the normalized singular value spectra of the normal and two staggered grids inversions. It is evident from these plots that a sharp cut-of on the singular value trend line occurs somewhere after the 270th singular value in all cases.

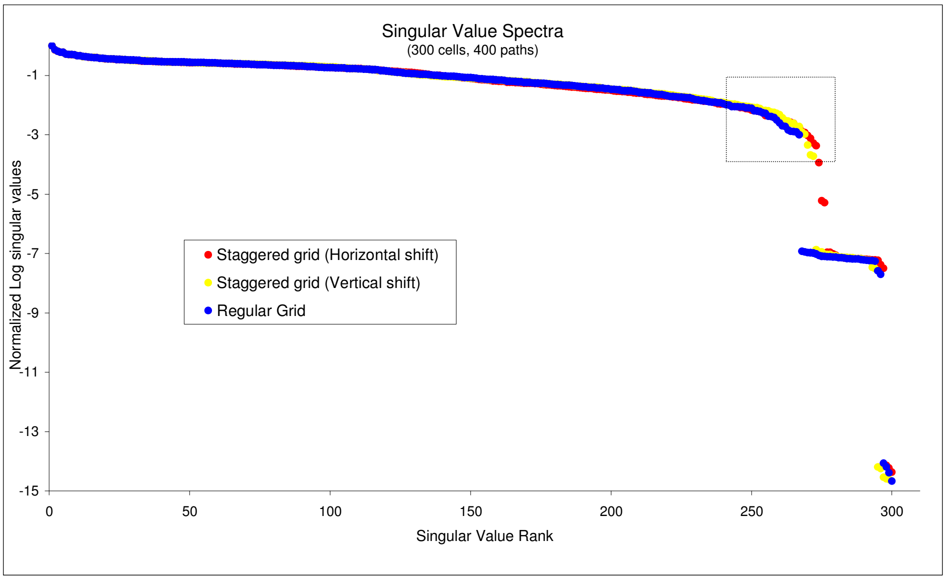

FIG. 4. Singular value spectra from the decomposition of normal and staggered grids.

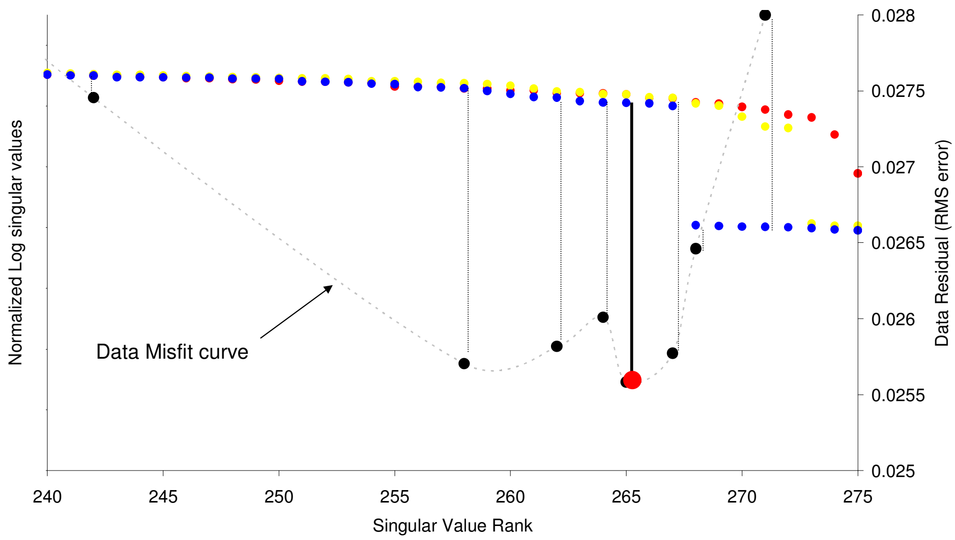

FIG. 5. Data misfit curve of TSVD inversions for selected singular values. Red dot indicates the threshold value with the lowest data RMS, chosen to carry out the numerical simulations.

Eight singular values in the vicinity of the abrupt change were tested as a lower band limit to carry out the TSVD inversion and the plot of the corresponding RMS data residual line versus the singular value spectra is shown in Figure 5.

In order to support the comparative tests between normal and staggered grid simulations, the velocity tomogram for noise free data is plotted in Figure 6a, where only in the lower right part of the model a higher velocity distribution than expected is recovered. The velocity model with the best recovery in parameter values, geometrical structures and the lower data errors for a normal grid inversion is shown in Figure 6b.

Grid shifting was performed in horizontal, vertical and diagonal directions by half a pixel step size length, resulting in summing up eight staggered grids for the final image. The same threshold singular value was selected for all tests, since, the singular values that correspond significantly to the solution, are almost the same when shifting the grid boundaries. The resulting tomograms of shifting the grid are shown in subsequent panels in Figure 6c and 6d.

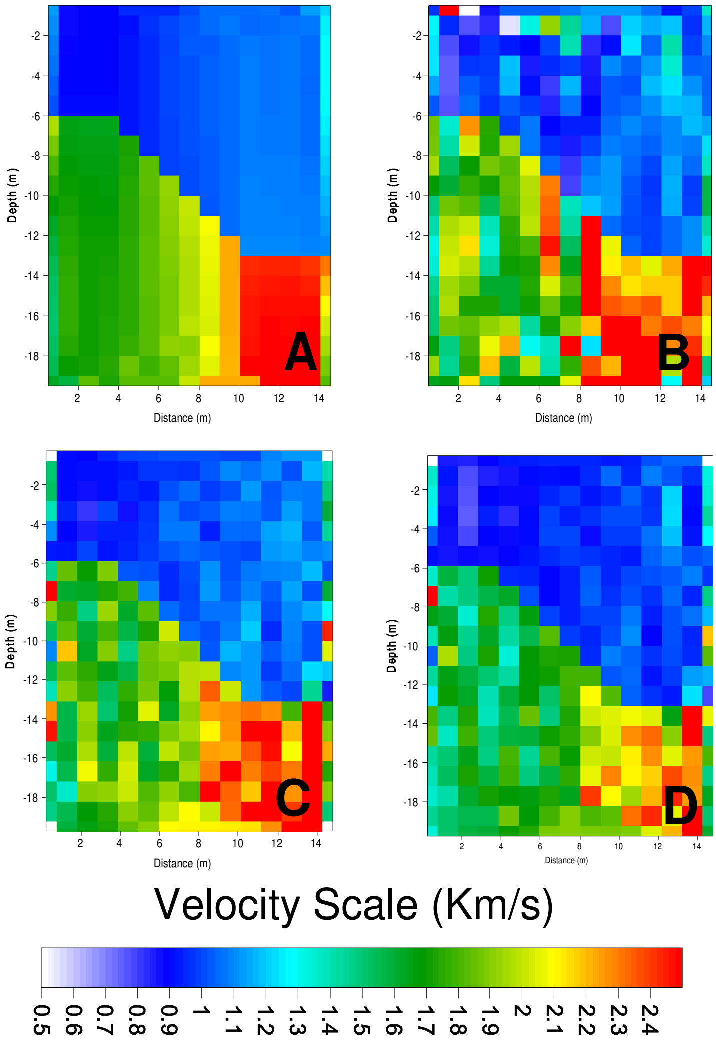

FIG. 6. Tomographic images of TSVD method for noise free data (a), noise contaminated Data using normal grid (b) and by incorporating four (c) and six (d) staggered regular grids

Figures 6c, 6d and Figure 7 were respectively obtained by staggering 4, 6 and 8 regular grids with a 15x20 pixel parameterization. Even though the velocity field of Figure 6c is significantly blurred, as a major result of the abrupt velocity contrast in the examined fault-like velocity structure and the noise added, the velocity pattern indicating the fault-like structure is clearly evident. The number of parameters for each shifted grid was kept the same as the normal grid in Figure 1; yet, the number of independent pixels used to plot the final tomogram is 8 times more resulting in a high-resolution final image. The trace of the fault shows good agreement between the tomographic images (Figures 6c, 6d, 7) and the desired target while velocity oscillations inside each staggered grid image become smoothed out as the number of staggered grids increases. The high velocity value at the lower right corner of the model that was observed even in the noisy free data case is less obvious at the final tomogram (Fig. 7) a fact that ensures the robustness of the examined grid manipulation method. The experiments carried out when shifting the grid boundaries show that local resolution can be better while noise effects can be smeared out significantly.

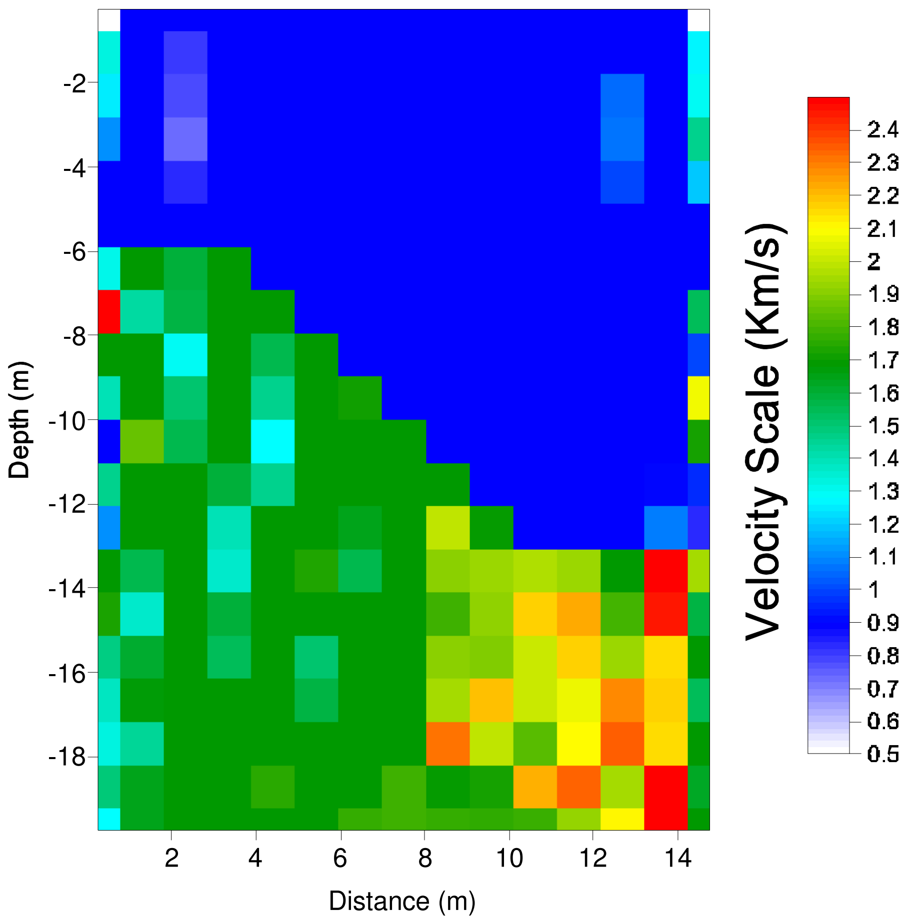

FIG. 7. Tomographic image obtained by incorporating 8 staggered regular grids.

In all of the reconstructed images utilizing the grid staggering technique, velocity restoration is superior to the normal grid approach whereas the data misfit remains almost at the same level. The above scheme of grid handling can be a robust mean in seismic tomography, especially in extreme velocity contrasts, that ensures stable results with minimum artifacts at the expense of repeated inversions.

CONCLUSIONS

Model parameterization holds the key in reducing the nonuniqueness in traveltime tomography. Since ray tracing is the most time-consuming part of travel time inversion, especially in high resolution grids, the implementation of staggered grids makes the above process computationally cheaper in most cases. Staggered grids can decrease the ambiguities and instabilities of the tomographic imaging. A method of grid manipulation as a viable way to treat the problem of uneven and poor ray coverage is implemented in a synthetic fault-like structure recovery. Noisy data are handled in an efficient way and the key features and velocity distribution of the fault like model are reconstructed with a satisfactory accuracy. The model variables of all staggered grids are superimposed, and each staggered image improves the local resolution recovering all possible information from the data without imposing constrains during the inversion or biasing the solution to a desired model. The accurate knowledge of seismic velocity distribution in the occurrence of abrupt discontinuities (faults or diapir boundaries). obtained by the implementation of staggered grids, can be highly desirable in geophysical or seismological applications, as for example in reflection seismic investigations (prestack depth migration) and earthquake relocation.

ACKNOWLEDGEMENTS

The present study was funded through the program EPEAEK II in the framework of the project “PYTHAGORAS II, Support of University research groups” with contract number 70/3/8023.

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