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Regional-residual magnetic field data separation in wavelet domain

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Abstract: *A new method for regional-residual magnetic field data separation, in wavelet domain, is suggested. The method is based on the discrete wavelet transform of either 1-D or 2-D data and the same transformation of an internal model for the regional field. The separation is controlled by the transformation of the model. The effectiveness of the method is demonstrated by application on synthetic 1-D and 2-D data. The results show an almost perfect separation of the two fields. A comparison with the polynomial fitting on synthetic data suggests that the suggested method produce better results. Finally the application of the technique to real data obtained from the exploration of an archaeological site in Northern Greece is presented.*

Keywords: *Magnetic field Separation, Discrete Wavelet Transform.*

INTRODUCTION

Total magnetic field data measured for geophysical exploration purposes comprise the superposition of the effects of all underground magnetic sources. Usually the targets in archaeological magnetic exploration are small, shallow depth anomalies, and their magnetic field is superimposed to the regional field that comes from larger or deeper sources. The estimation and subtraction of the regional field leads to the residual field that corresponds to the target sources. Evidently, the reliability of the interpretation of the residual field depends on the correct estimation of the regional field.

All regional-residual field separation methods are based on the fact that the regional field is smooth and its spectrum is dominated by relatively low frequencies. The most common regional-residual separation methods fall in one of the next categories:

1) Empirical graphical methods and methods of smoothing of equipotential curves of magnetic induction (Agocs, 1951; Li and Oldenburg, 1998). These methods are subjective and depended on the interpreters' experience.

2) Polynomial regression: The regional field is modeled with a first or second order polynomial and the residual field is regarded as an error between the model and the data. The coefficients of the polynomial are calculated with least squares. Polynomial regression is the most popular method (Tsokas et al., 1995).
3) Digital filtering in frequency domain or in spatial domain in order to separate the low-frequency regional anomalies from the higher frequencies that correspond to the residual field (Griffin, 1949; Zurfueh, 1967). Band pass and Wiener optimum filters are referred also (Pawlowsky and Hansen, 1990). The specification of the transfer function of a Wiener filter is subjective and requires a reference field and its spectral power. Generally, the spectral separation gives poor results because of the spectral overlap of the two fields.
4) Data transformations and enhancement of one field component against the other. For example upward continuation attenuates anomalies caused by local, shallow sources relative to anomalies caused by deeper sources (Blakely, 1995). On the contrary downward continuation amplifies the residual field with respect to the regional one. Li and Oldenburg (1998) presented a method of regional-residual field separation that can be regarded as 3-D multiscale inversion. The advantage of the method is that doesn't affect the shape of the anomaly and is independent of spectral overlap, but its application requires specification of a number of parameters.
5) Multiscale wavelet analysis. The wavelet transform-based multiscale analysis helps to discard undesired components of the signal (Mallat, 1989; Daubechies, 1990; Mayer, 1993; Kaiser, 1994). A typical wavelet domain detrending is twofold (Fleming, 2000):

- Wavelet transformation of the data and adjustment of the decomposition level until the isolation of the trend
- Setting the corresponding wavelet coefficients to zero (or compressing them) and reconstruction of the original signal minus the trend.

Fedi and Quarta (2000) describe a similar regional-residual separation method in discrete wavelet domain. They look for a wavelet basis that gives a compact transformation of the data. Since that analysis in not shift invariant they do the same for a number of signal shifts and they finally choose the combination of wavelet and signal shift that gives the most compact transformation. The criterion of minimum entropy is used for the choice of the most compact transformation. They use a gravity synthetic example and achieve an almost perfect separation. Ucan et al. (2000) presented a scheme for multi-resolution analysis of 2-D data and the isolation of adjacent anomalies. They perform one level of 2-D multi-resolution analysis of data and keep the coarse image.

In general, the isolation of the regional field is a difficult problem because of the spectral overlapping with the residual field. The compression of the coefficients is also a problem, since the reduction to zero of all coefficients of a level degrades the signal. The proposed new scheme for the regional-residual separation in wavelet domain includes an internal model for the regional field that controls the separation.

DESCRIPTION OF THE PROPOSED ALGORITHM

The proposed method is based on the fact that compact discrete wavelets transform of a first or second order polynomial has exactly the same structure independent of coefficients of the polynomial. That means that all first order fields have the same null coefficients in their wavelet transformation. So we can estimate at first the regional field following the next steps:

1. Choice of a model for the regional field (first or second order polynomial).
2. Selection of the proper wavelet that gives the most compact discrete wavelet transform of the model.
3. Wavelet transformation of the measured data and the model.
4. Comparison of the two transformations and reduction to zero of the data coefficients corresponding to null coefficients of the model.
5. Reconstruction of the signal that mostly corresponds to the regional field.
6. Because of the spectral overlapping the reconstructed signal comprises a part of the residual field. Thus, a new wavelet basis is selected and steps 3and 4 are repeated.
7. For better results, we repeat the algorithm as many times as we wish.
8. Finally we subtract the regional field from the original data.

The block diagram of Figure 1 shows the structure of the total schema. In 2-D case we use 2-D wavelet transform(Mallat, 1998) and 2-D model (a level or a second order surface). The algorithm is implemented in Matlab with the aid of Wavelab802, a free offered wavelet Matlab toolbox by the Stanford University.

CHOICE OF THE MODEL AND WAVELET

The choice of the model is empirical depended on the interpreter's experience. We tried a number of wavelets orthogonal or biorthogonal and found that the Daubechies wavelet with 4 coefficients gives a very compact transformation for the first order model. Only 9 from 32 wavelet coefficients are non-zero. The biorthogonal triangular wavelet gives an equally compact transformation. Thus a good choice of wavelet pairs for the first order model is the Db4 and triangular biorthogonal wavelet. For the second order model we used the Db6 and Villasenor1 wavelets. The coefficients of the corresponding digital filters are listed in Table 1.

1-D synthetic examples

We illustrate the efficiency of the method with four 1-D synthetic examples. The basic model for the synthetic residual field is the total field anomaly of an orthogonal prismatic body originated from the subtraction of two magnetized vertical sided prisms of infinite depth extent.

According to Logacev and Zaharov (1973) the total field anomaly over a vertical plate with infinite horizontal length and vertical extend is

$$\Delta T(x) = 4bJ\frac{\cos^2 In}{H^2+x^2}\cdot \cdot\begin{Bmatrix} H(\tan^2(In)-\cos^2(D))- \\ -2x\tan(In)\cos(D) \end{Bmatrix} \qquad (1)$$

where, J is the magnetization, In is the inclination angle of the normal magnetic field, D is the azimuth of the measuring profile. The profile is perpendicular to

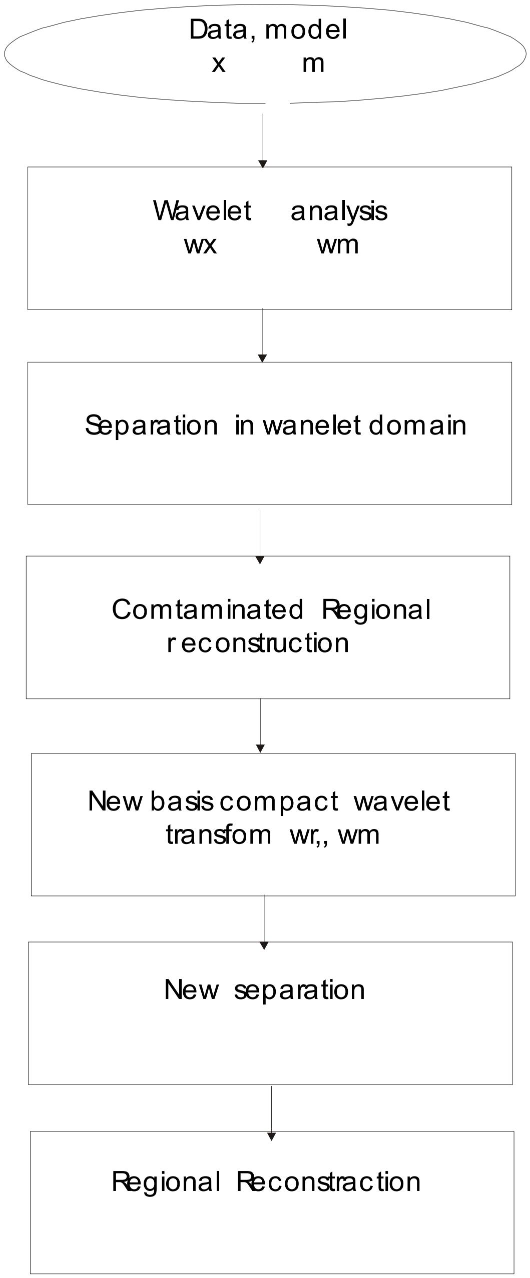

Fig.1. Block diagram of the proposed algorithm.

the elongation axis of the prism, H is the burial depth of the upper surface of the structure, 2b is the width of the plate and x is the distance from the prism's epicenter. In the case of an orthogonal prism produced by subtraction of two plates at depths H_1 and H_2, as in Figure 2, the total field is given by

$\Delta T = \Delta T_1 - \Delta T_2$. (2)

We selected a total profile length of 32 m, and a sampling interval of 1 m. The width of the prism, A, is one sampling interval. For the regional field we use the model:

$y = 2 + 1.2x$.

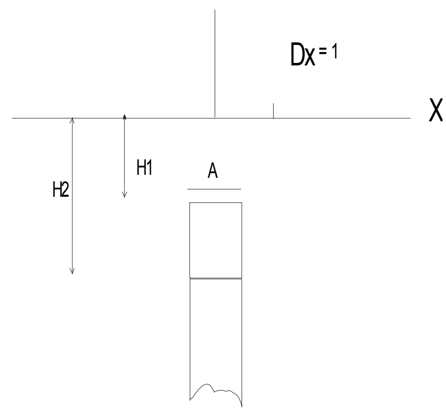

Fig.2. Model for the residual field

The three synthetic fields, residual, regional and total, are shown in Figure 3. Application of the proposed approach to the total synthetic field gives a very good estimation of the regional field. If we repeat several times the algorithm we achieve an almost perfect coincidence of the estimated the actual regional field as demonstrated in Figure 4.

As a second example we use the same residual field and replace the regional field with a second order curve,

$y = 2 + 0.4x + 0.2x^2$.

We also replace the previous used wavelets by the Daubechies 6 and

Villasenor (the corresponding coefficients are listed in Table 1) and repeat the procedure. The results are shown in Figure 5.

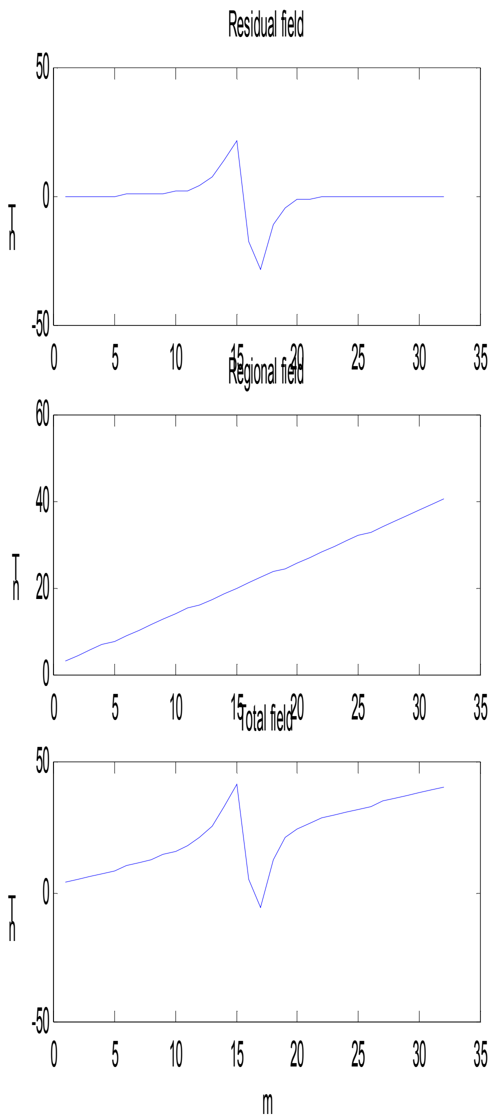

Fig.3. Synthetic residual field in the upper part, the used regional in the middle and the field obtained from the superposition of those in the lower part.

As we see in Figure 6 we achieve a quite good estimation of the regional field, using a first order model for the estimation of a second order regional field. This is due to the ability of wavelets to concentrate the signal's energy in a few wavelet coefficients. Thus the objectiveness of the model selection has little effect in the efficiency of the method.

2-D synthetic example

In 2-D case a initial signal of a magnetized prismatic body is corrupted by adding noise (std=3) and a first order 2-D regional field (Fig. 7). Application of the algorithm gives an estimation of the regional field with a mean error of 0.2511 nT. The truth regional field and the estimated one are shown in Figure 8.

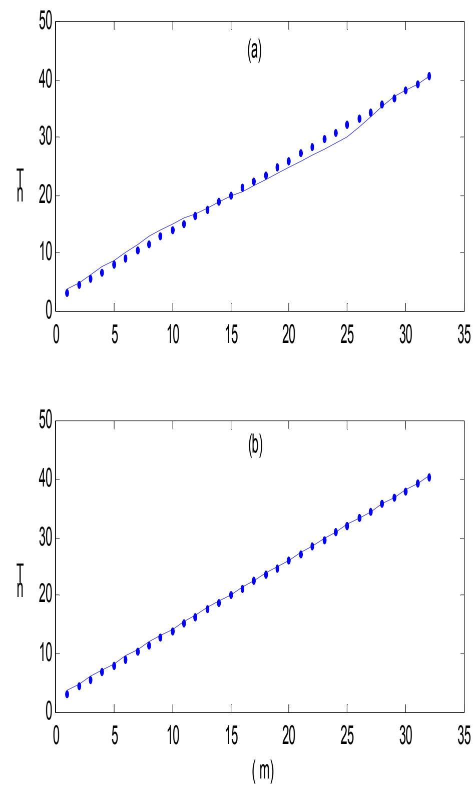

Fig.4. Coincidence of the estimated with the truth regional field: a) application of the algorithm once. b) repeated application of the algorithm. The dot line depicts the truth regional field while the solid one the estimated regional field

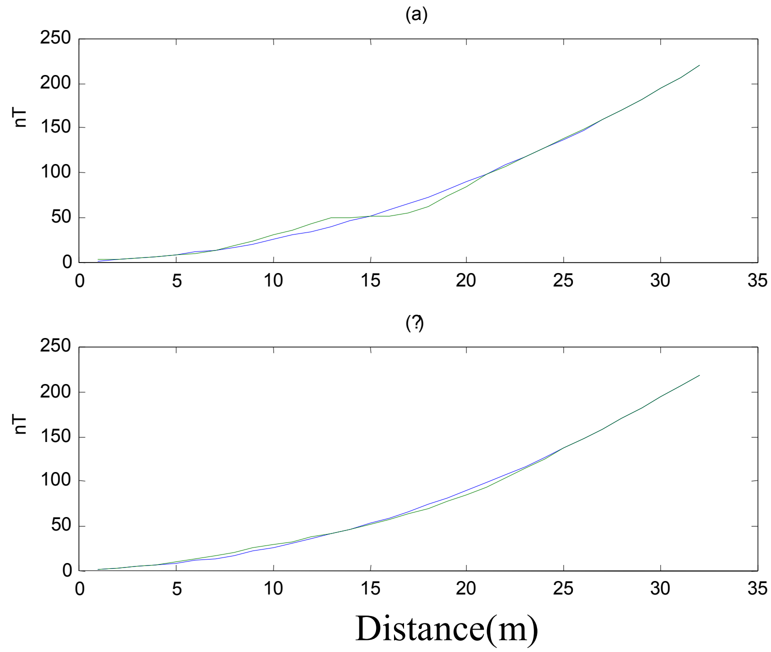

Fig.5. Coincidence of the second order estimated field with the actual regional field: a) Application of the algorithm once. b) repeated application of the algorithm. The blue line depicts the actual regional field while the green one is the estimated regional field.

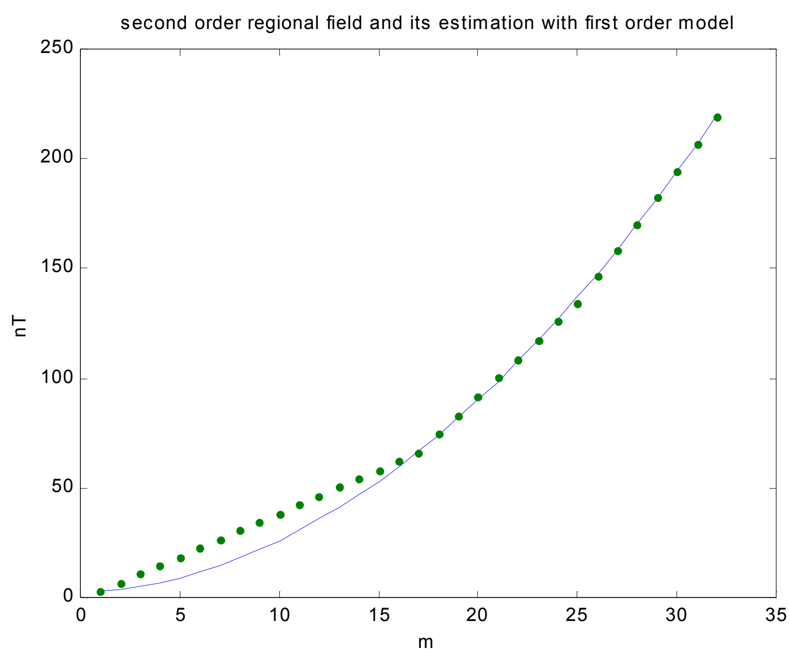

Fig.6. Coincidence of the estimated (dot curve) second order regional field with the truth regional field (solid curve) in case of wrong model order selection

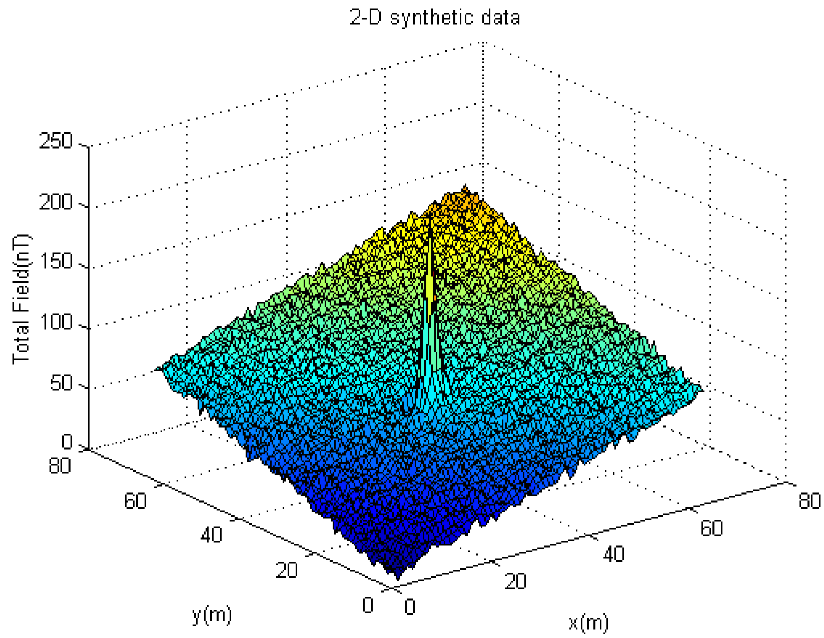

Fig.7. 2-D Synthetic magnetic data of a prismatic body with addition of noise and a first order regional field

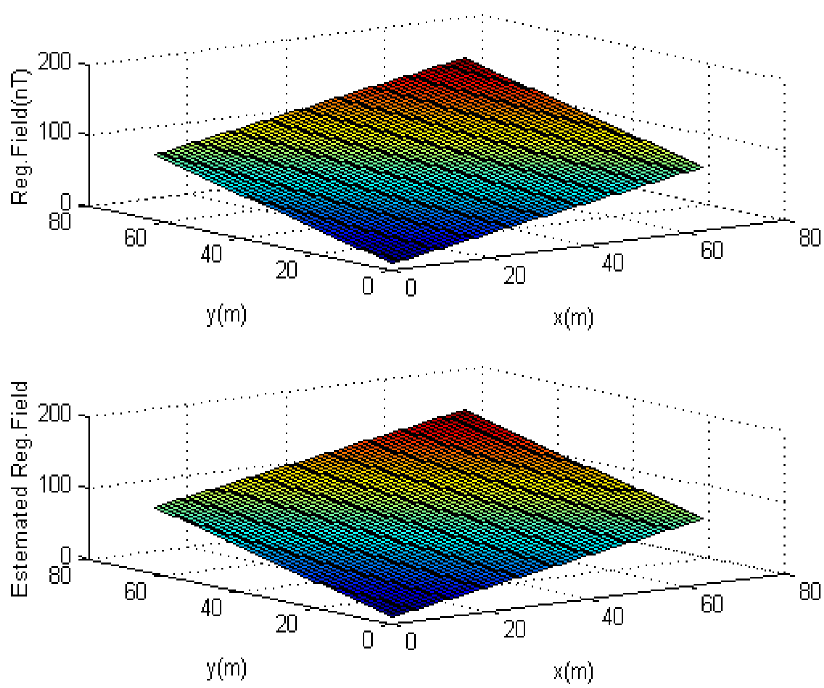

Fig.8. Up: synthetic regional field down: estimated regional field

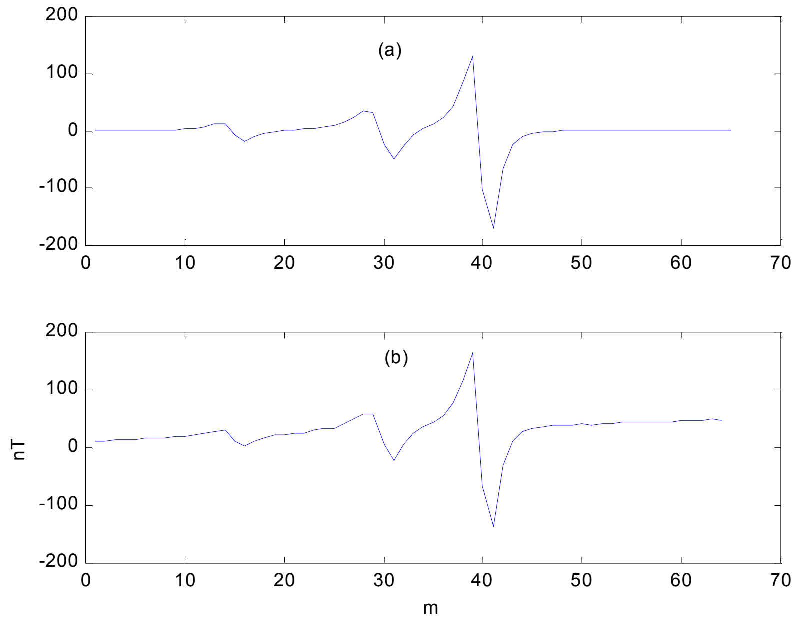

Fig.9. a) Anomaly field of three vertical prismatic bodies at positions 15,30,40 m of a profile with 64 m length.
b) The same field with the addition of random noise and a synthetic regional field

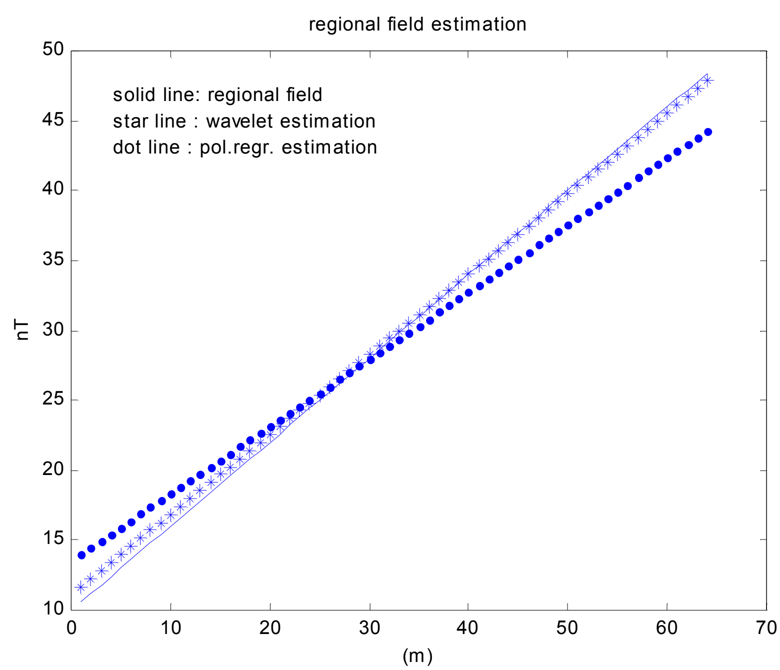

Fig.10. Linear regional field (solid line). Its wavelet estimation (star line) and the polynomial reg. Estimation (dot line).

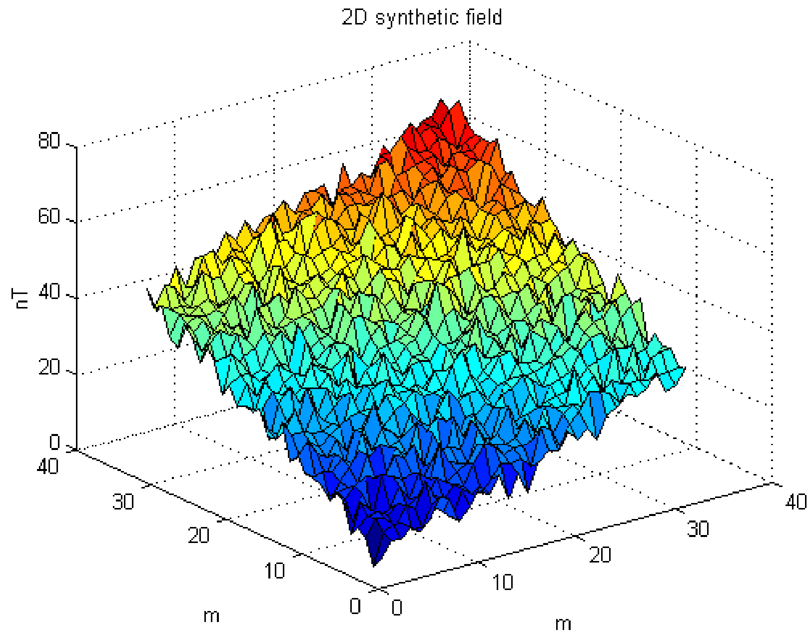

Fig.11. 2D synthetic field consisting from a level surface and a random noise

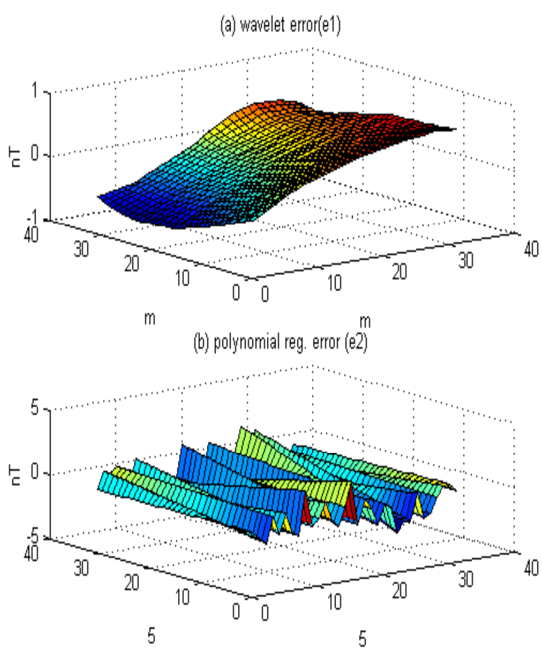

Fig.12. Synthetic plane estimation errors: a) wavelet method error, b) polynomial regression error

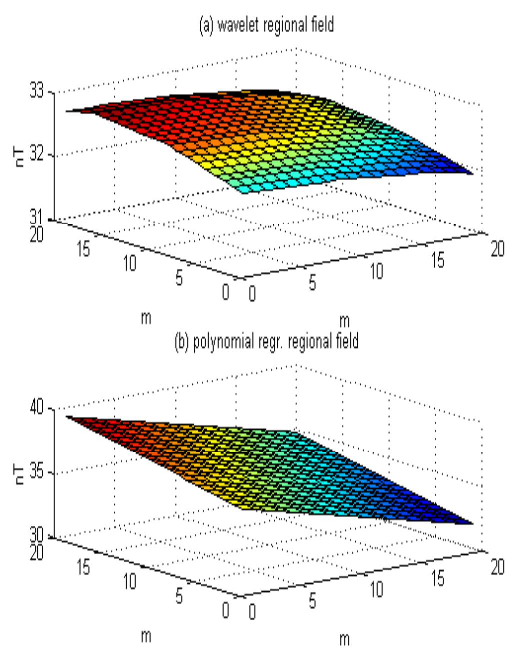

Fig.13. a)wavelet regional field, b) polynomial regression regional field

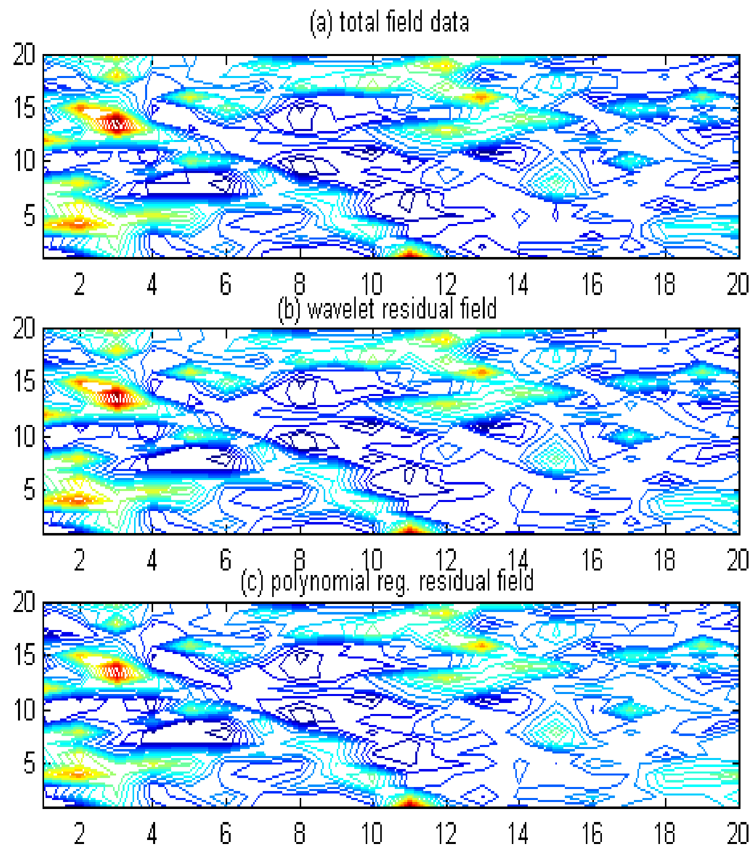

Fig.14. a) Total field data from the archaeological site Europos in Northern Greece b) Wavelet residual field, c) Polynomial reg. residual field

COMPARISON WITH POLYNOMIAL REGRESSION

For comparison with polynomial regression we use a model of three prisms along a profile length 64 meters in positions 15, 30 and 40 m. The first two prisms are 1x1 vertical prismatic bodies (0.003 nT susceptibility contrast) at depth 1.5 m, and the third is a 2x1 prismatic body with the same susceptibility contrast, at the depth 1m. Figure 9a shows the total field anomaly of this model.

For the linear trend we use the model f=10+8.6*x contaminated with a random noise (std=1). The overall synthetic data are shown in Figure 9b. The norm of the regional field (solid line in Figure10) is 252.1079 nT. The norm of its wavelet estimation (star line in Figure 10) is 252.5594 nT. The polynomial regression gives an estimation of the regional field (dot line in Figure 10) with norm 280.7514 nT. The relative poor estimation that is given by the polynomial method, is due to the fact that polynomial regression estimates the best line through the data, even though the original residual field may be not symmetrical about zero.

In 2D case we use the synthetic field of Figure 11. This field consists of a 32x32 m level(x=5+0.9*x1+1.3*x2) corrupted with random noise (std=3). The application of the wavelet method gives an estimation of the level surface with an error e1(Fig. 12a).The norm of e1 is 12.1568 nT. The error of the polynomial regression e2 (Fig. 12b) has a norm of 17.8017 nT. From this example we conclude that even in the case of a level with noise where the polynomial regression gives the most reliable results, the wavelet method approaches better the level surface.

REAL DATA EXAMPLE

Regional-residual magnetic field separation has been applied to a real data set ontained from the exploration of archaeological site Europos in Northern Greece (acropolis). Europos was a commercial center on the banks of the river Axios in Northern Greece. The ruins of the ancient urban center and installations are hosted in the subsurface, near the village that bears the same name.

The data used for regional-residual separation were obtained from the exploration of the cemetery of the Roman Era. The application of the new wavelet method and the polynomial regression gave the results shown in Figure 13.

We observe that even though we used a linear model for the regional field, the wavelet method finds a surface with a small curvature instead of a level. Figure 14 shows the contour map of the original total field magnetic data, and the estimation of residual field by the wavelet method and by the polynomial regression.

CONCLUSIONS

The proposed new regional – residual magnetic field data separation method is simple, accurate, and achieves a very good isolation of the regional field in the wavelet domain. It is independent of the nature of the field and can be used in all field data types, 1-D or 2-D.

ORTHOGONAL				**BIORTHOGONAL**			
Db 4		**Db 6**		**Triangle**		**Villasenor 1**	
qmf	**dqmf**	**qmf**	**dqmf**	**qmf**	**dqmf**	**qmf**	**dqmf**
0.4830	0.4830	0.3327	0.3327	0	0.5	0.0378	-0.0645
0.8365	0.8365	0.8069	0.8069	1	1	-0.0238	-0.0407
0.2241	0.2241	0.4599	0.4599	0	0.5	-0.1106	0.4181
-0.1294	-0.1294	-0.1350	-0.1350			0.3774	0.7885
		-0.0854	-0.0854			0.8527	0.4181
		0.0352	0.0352			0.3774	-0.0407
						-0.1106	-0.0645
						-0.0238	
						0.0378	

TABLE 1. Coefficients of the used wavelet filters. (qmf depicts the analysis filters ,dqmf depicts the synthesis filters)

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