Automated mesh design for two-dimensional magnetotelluric interpretation codes

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Abstract: A scheme and associated computer programme for designing input mesh for two-dimensional (2D) magnetotelluric (MT) inversion codes are presented. The scheme is based on the so-called penetration depth (PD) concept. A mesh is designed using the average values for the smallest and the largest PDs of all stations along the profile. The location of additional rows and columns required by boundary condition applications are also printed out in a separate file in order to be fixed by the inversion code. An illustrative example of automatic mesh generation is presented. The scheme can be applied to other geoelectrical methods.

INTRODUCTION

In two-dimensional (2D) MT inversion for subsurface resistivity distribution (e.g. Jupp and Vozoff, 1977; deLugao *et al.*, 1997; Mackie *et al.*, 1997), the accuracy of the solution depends on both the data and the mesh used to represent the subsurface geology. The mesh design is usually based on the available a priori information and experience. In the currently popular 2D MT inversion codes such as the OCCAM (deGroot-Hedlin and Constable, 1990), RRI (Smith and Booker, 1991) and NLCG (Mackie *et al.*, 1997), the mesh is designed by the user. This process is time-consuming and sometimes incurs user-dependent errors. The grid size is very much dependent on the frequency of the field variations because the dimensions of the grid cells must be kept small compared with the skin depth in order that the numerical solution approximation may be generally valid. For this reason, portable inversion codes ideally include an automatic mesh generator which is not only a great time-saver but also an eliminator for user-related errors (Weaver, 1994).

The information obtained from the data via simple process such as Penetration depth (PD) and Niblett-Bostick transformation, NBT (Niblett and Say-Wittgenstein, 1960; Bostick 1977; Jones, 1983), may be used to create a mesh that represent the subsurface when no prior information is available. The PD and NBT are common and easy-to-use tools for quick interpretation of electrical and electromagnetic (EM) data.

Note that the difference between PD and well-known skin depth definitions is that the attenuation factor for the EM wave in a half space medium of resistivity equal to the apparent resistivity is ½ rather than 1/e (Jones, 1983). It is thus clear that skin depth is deeper than PD. From both definitions one may estimate the depth above which the EM wave sweeps most of the information to surface. As a result, the inversion schemes cannot resolve any geo-electrical structure below the PD because of high attenuation of the EM wave. That is why we will adopt the PD formulation for automatic mesh generation in this paper and we will focus on the case where the earth is 2D in structure (i.e., the PD is different at each station since resistivity varies in lateral and vertical directions along the profile).

Using a similar concept, Oldenburg and Li (1999) show that the inversion process is unable to produce meaningful model below the depth of investigation (DOI in their paper) and proposed a scheme to delineate the depth level below which the model parameters are not constrained by the data. However, the modeling theory requires the mesh to extend deeper than that level in order to satisfy boundary conditions. In practice, 8 to 10 rows and columns of the cells are usually added to the mesh depending on the code and speed of computers. In the inversion process the added cells will only increase the number of unknown parameters.

(a)

(b)

i) nx=28

100000, 20000, 5000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000,

1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 5000, 20000, 100000

ii) nz=18

25, 25, 25, 25, 50, 100, 50, 100, 200, 500, 500, 1000, 1000, 2500, 4000, 9000, 25000, 100000

FIG. 1. (a) Simple 2D model consist of 2 layers (100 and 1 Ohm.m) underlain by a basement of 500 Ohm.m. The model contains a low resistivity (5 Ohm.m) block imbedded in top layer. Arrows indicate the position of the stations. (b) Width (i) and thickness (ii) of the cells used for calculation.

The data-dependent mesh design (DDMD) approach may help the user to overcome the problems highlighted above. Using the PD concept, the interpreter may create a mesh and mark out the region, the parameters inside of which are constrained by the data and the parameters outside of which may be fixed for any appropriate value to fulfill the boundary conditions. The user should be aware that DDMD does not remove the boundary conditions but prevents unnecessary number of cells that, in turn, would increase the size of both the model and calculation time.

MT METHOD

The EM wave impedance, better known as the Cagniard (1953); Tikhonov (1950) impedance, is defined as

$$Z = E_x / H_y \text{ or } E_y / H_x , \qquad (1)$$

where E and H are the electric and magnetic field components of the EM wave respectively, while the indices x and y indicate the measurement directions of the related field. The impedance (Z) values are assigned to xy or yx direction representing the Transverse Electric (TE) or Transverse Magnetic (TM) polarization mode depending upon the extension of the geological target and measurement direction. The apparent resistivity (ρ_a) and phase of impedance (Φ_z) data are usually adopted as the MT data for inversion and are given by

$$\rho_{aij} = (\omega\mu)^{-1/2} |Z_{ij}|^2, \quad ij = xy, yx \qquad (2)$$

and

$$\Phi_{zij} = \tan^{-1} (\text{Imaginary} (Z_{ij}))/\text{Real} (Z_{ij})), \quad ij=xy, yx. \qquad (3)$$

For each MT station, the observed ρ_a and Φ_z values for a range of frequencies in two perpendicular directions (i.e., TE and TM modes) are interpreted in 2D inversion. Note that if the rotation angle is taken as an interpretative parameter, then the xx and yy values can also be incorporated to the inversion process (deGroot-Hedlin, 1995) but it is not routine practice. The PD for MT method is approximated by

$$D_{ij} = (\rho_{aij} /\omega\mu)^{-1/2}, ij = xy, yx . \qquad (4)$$

2D MODELING PROCESS

The fundamental equations for 2D modeling are well documented in the literature (e.g. Brewit-Taylor, 1976; Zhdanov *et al*., 1982; Wannamaker *et al*., 1985) and will not be re-counted here. The inversion codes based on these or similar modeling schemes are also widely in use (e.g. Sasaki, 1989; Uchida and Ogawa, 1993; de Lugão et al., 1997; Mackie et al., 1997). All these codes necessarily require inputs such as field responses, mesh specifications, and stopping and stabilization criteria for inversion but in different formats. The observed data for all the stations along the profile are given sequentially and initial resistivity model consists of a mesh of cells each of with defined width, thickness and resistivity. The thickness and width of each cell are crucial parameters controlling the accuracy of the solution. The effect of these parameters may be shown by the response of the simple model given in Figure 1a. The model has 2 layers (100 and 250 m thick) underlain by crystalline basement. The resistivities of the layers and basement are 100, 1 and 500 Ohm.m, respectively, and a block of 5 Ohm.m is imbedded into the top layer. The MT responses for two different meshes have been calculated using the well-known 2-D finite element modelling code (PW2D) of Wannamaker *et al*. (1985) and a code based on finite differences modelling scheme developed by one of us (MEC) for illustration.

The first grid design used for model computation (Mesh 1) consists of 28x18 cells. Four rows of cells, each 25 m thick, comprise the first geoelectric layer as shown in the upper part of Figure 1b. The second mesh (Mesh 2) consisting of 28x16 cells is different in that only two rows make up the first layer and the thickness of each row is 50 m. The widths of the cells for both meshes have been kept same as those are shown in lower part of Figure 1b. Both meshes have 8 rows of air layers for the TE mode calculations. The forward calculation has been performed for 17 frequencies ranging from 8192 to 0.125 Hz.

The TE and TM mode responses of this model at selected point on the model (stations S1 and S2) are presented as apparent resistivity and phase of impedance in Figure 2. The red lines show the response values from Mesh 1 while blue ones are for Mesh 2. Considering that the only difference between these two meshes is the number of rows used to represent the top layer and the embedded 2D body, the different results and the erroneous responses emphasize the need for accurate grid design in MT forward modeling and inversion. Note that although Mesh 1 gives a more stable result, it still has some design error due to incorrect ratio of the thickness and the width for the cells at upper rows. This error can occur when a user intends opts for

reducing the number of blocks in the mesh in order to have smaller mesh and speed up the inversion process.

The DDMD approach adopted in this paper may help overcome this problem by handling all stations along the profile and creating an optimum mesh that is more appropriate for the requirements of all stations. Wannamaker et al. (1985) and Weaver (1994) give an extensive guideline for better mesh design. The following scheme is based on those guidelines but is different in the sense that we use the PD rather than skin depth definition.

a) nx = 89				
1216418.	243284.	48657.	9731.	1946.
487.	162.	168.	503.	168.
162.	168.	503.	168.	162.
168.	503.	168.	162.	168.
503.	168.	162.	168.	503.
168.	162.	168.	503.	168.
162.	168.	503.	168.	162.
168.	503.	168.	162.	168.
503.	168.	162.	168.	503.
168.	162.	168.	503.	168.
162.	168.	503.	168.	162.
168.	503.	168.	162.	168.
503.	168.	162.	168.	503.
168.	162.	168.	503.	168.
162.	168.	503.	168.	162.
168.	503.	168.	162.	168.
503.	168.	162.	487.	1946.
9731.	48657.	243284.	1216418.	
b) nz = 31				
11.	13.	15.	17.	20.
20.	20.	20.	20.	20.
20.	20.	20.	21.	21.
21.	21.	22.	22.	22.
22.	22.	23.	23.	23.
23.	70.	210.	629.	1887.
9433.				

FIG.4. (a) Width and (b) Thickness of the cells calculated by the programme using the data obtained from first mesh of the model given in Fig. 1a as observable data.

ALGORITHM

Assuming that the input data for a 2D modeling or inversion code is as in Figure 3 (x values are the distance from the first station, z values are the topographic height
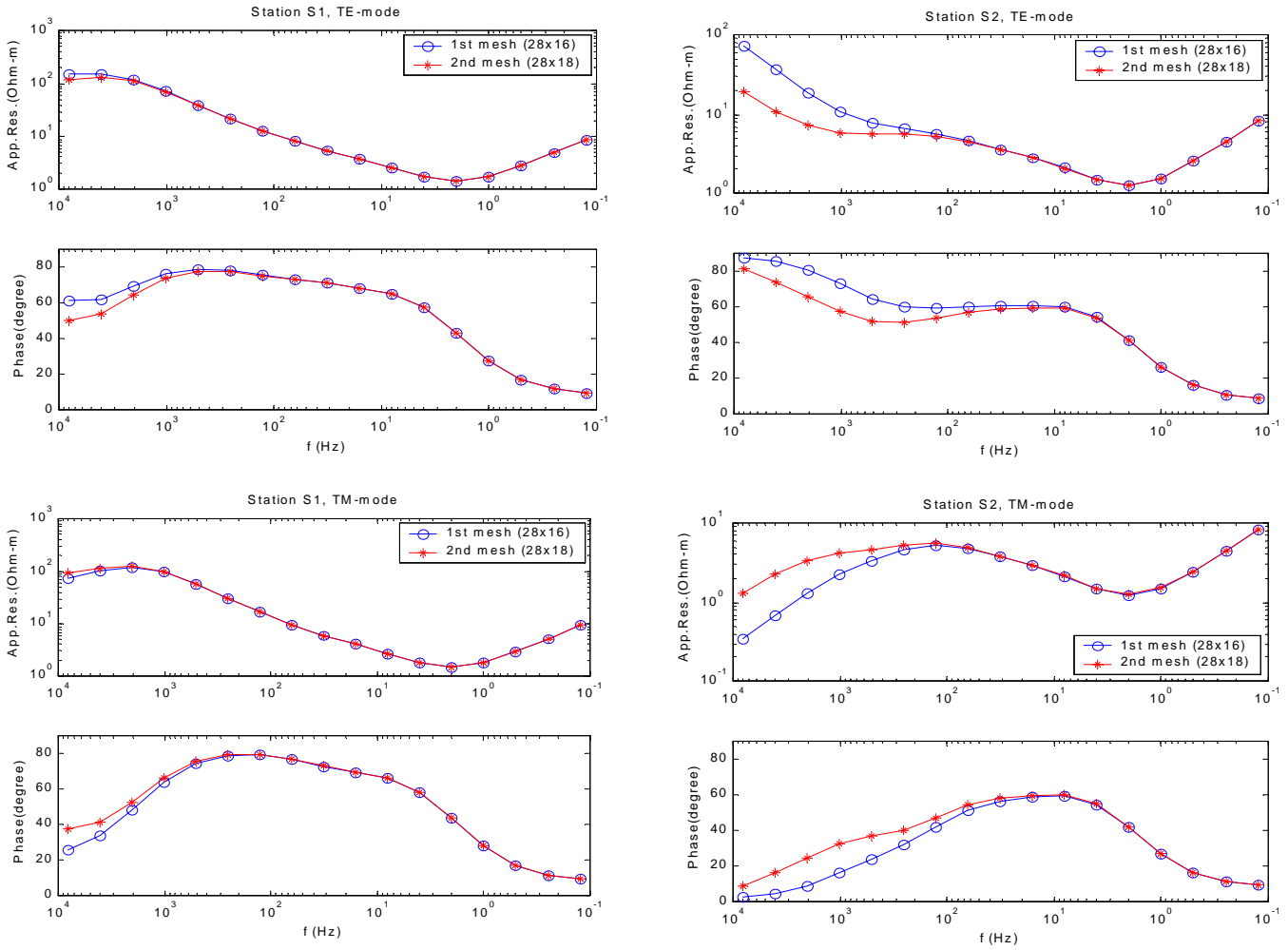
FIG. 2. The TE and TM mode responses of this model at station S1 and S2 as apparent resistivity and phase of impedance.

```
Title                   ! project or profile title
2                       ! number of station
5                       !# of frequencies in following station
 0 1 1                  !  x, z, external factor for static shift
.10 10 1 45 1           !frequency, apparent resistivity, error Phase of impedance, error
1.0 20 1 40 1
10  30 1 35 1
100 10 1 30 1
100 1.0 1 35 1
4                       !# of frequencies in following station
10 1  1                 !  x ,z, external factor for static shift
.10 10 1 45 1

1.0 20 1 40 1
10  30 1 35 1
100 10 1 30 1
```

FIG. 3. Sample input for 2D inversion code and mesh design programme.

from the datum level; all units are in meters), then an explicit algorithm can be written as

1 read the profile data

2 calculate the lowest PDs for each station

3 remove the maximum and minimum values from the list, calculate average value and divide it by 3 for the minimum thickness

4 calculate the largest PDs for each station

5 remove the maximum and minimum values from the list and calculate average value for the maximum thickness

6 calculate the topographic height differences and, if necessary, adjust the minimum thickness value

7 calculate distance between stations and arrange the number of cell which will be placed between them

8 print out the mesh and list of the cells to be fixed.

A programme (DDMD) has been written based on the above algorithm. The assigned cell resistivities are all equal in value so that it can be used as a homogeneous half space initial guess model in an inversion process. A list of fixed parameters may be produced and these parameters may then be excluded from an inversion process. The programme currently produces 2 different meshes, which can be read by either rund2dinv_nlcg (Mackie et al., 1997) or Uchida and Ogawa's code (1993). The input and output can easily be modified for any kind of data and mesh format.

SAMPLE APPLICATIONS TO SYNTHETIC DATA

As an illustrative example, we have applied DDMD to the data obtained from Mesh 1 for the model given in Figure 1a to produce an optimum mesh necessary for an inversion process. The resultant mesh has 89x31 cells and the width (a) and thickness (b) are shown in Figure 4. To be able to compare the result, a different mesh obtained from mesh generator of WinGlink™ is given in Figure 5. The last one (32x31) is produced by using "Coarse" option with a resistivity of 100 Ohm.m for homogeneous half space where thickness of the rows is increasing according to a pre-assigned fixed scheme, namely 1.5 and 1.2 for horizontal and vertical, respectively (WinGLink, 2000). Note that if one uses "Fine" option for same homogeneous half space, number of the cells in the produced mesh will be 481x73 with minimum distances in x and z are 3m and 1m, respectively. Resistivity of homogeneous half space plays a key role in these options. The lower the value is

given the larger the number of cells the programme produces. Selection of the resistivity value for homogeneous half space could be cumbersome in the case of complex geological environments. As a mater of fact, the DDMD scheme is equivalent to select minimum resistivity value among the shallow values and use it to calculate a starting level and delineate the area according to the largest depth obtained from the resistivity value among the lover frequencies. Thus, classical approaches use a single value while DDMD uses double value to create a mesh. The number of the cells that DDMD produces may seem needlessly large as initial mesh used to generate this test data had only 28x18 cells. However, in practical situations, the geometry of the subsurface target is not known in advance and DDMD generates the optimum size of the cells that are justified by the data enabling the inversion schemes to simulate accurate model responses. Note that this definition should not be mixed with the minimum structure inversion (e.g. deGroot-Hedlin and Constable, 1990). The cells produced by DDMD are not the structure but will serve to construct the subsurface structure. Result of the inversion codes have been avoided due to fact that it also requires to judge the solution power of the inversion schemes which remain out of frame of this work.

CONCLUSIONS

The programme DDMD is an easy-to-use scheme for creating mesh for inversion process. It does not eliminate boundary condition problem of modeling theory but delineates the region over which the modeling scheme may produce a meaningful result. Also it lists the cells that may be fixed to any appropriate values and excluded from the inversion process.

It can be modified to handle a priori information such as the results of 1D inversion but these features have not yet been included to the current version due to the fact that there is no standard input format for the existing 2D MT codes (e.g. see Wannamaker et al., 1985; Mackie et al., 1997). Although it produces a large number of blocks, it can be shown that these are constrained by the data. Although not shown here, the DDMD code has been tested on a number of field data and the meshes have been used to produce geo-electrical models with various inversion programmes. In particular, we have obtained satisfactory results for field data when the output of DDMD is subsequently inverted using a 2D inversion code written by one of us (Ulugergerli and Meju, 1997) and rund2dinv_nlcg (Mackie et al., 1997).

```
nx=32
100000  30000  10000   3000   1000   1000   1000   1000   1000   1000
  1000   1000   1000   1000   1000   1000   1000   1000   1000   1000
  1000   1000   1000   1000   1000   1000   1000   1000   3000  10000
 30000 100000


nz=31
    25     50    100    100    100    100    100    100    200    200
   200    200    200    200    200    200    200    200    500    500
   500    750   1250   2000   3000   4500   7500  10000  15000  25000
 50000
```

FIG. 5. (a) Width. and (b) Thickness of the cells calculated by WinGLink™ using the data obtained from first mesh of the model given in Fig.1a as observable data.

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