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A discrete Markov model for earthquake occurrences in Southern Alaska and Aleutian Islands

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Abstract: *A large number of stochastic models are currently available for the earthquake occurrence. The Markov model is applied to data from the area of southern Alaska (peninsula of Alaska and Shumagin islands) and the Aleutian Islands in order to investigate for great earthquake occurrence in space and time. The model defines a process in which successive state occupancies are governed by the transition probabilities of the Markov process. Each element* p_{ij} *, of the transition probability matrix represents the probability that the state is j at the time t+1, given that the state was i at the time t, and the probability can be written as:* $p_{ij}=Prob[X_{t+1}=j \mid X_t=i]$*. The area of southern Alaska and Aleutian Islands is divided in three seismic zones that are defined as states in the present study. Thus the earthquakes, which migrate from zone to zone, i.e. from state to state, carry with them the number of the zone in which they occurred. In this way we can examine the genesis of the earthquakes in the investigated area in a quantitative way, through the transition probabilities of the defined process. A pattern for an east-west migration, in space and in time, of large (*$M \geq 7.0$*) earthquakes is found. A two-state Markov model is applied in the three zones, which suggests periods of activity and quiescence. The application of this model makes it possible to establish whether in a specific time period a state is in an active or an inactive period and this is useful for seismic hazard analysis.*

Key Words: *Markov Model, Transition Probability, East-west Pattern, Alaska and Aleutian Islands*.

INTRODUCTION AND DATA USED

In the attempt to overcome the modelling problems associated with the memoriless property of the Poisson process, other stochastic models have been considered. Markov models are useful in describing a unique type of dependence in a sequence of events. For this model a state space $E=\{1,2,3,...,N\}$ is defined. The process $\{X(t), t \geq 0\}$ describes the visits to these states and is said to be a Markov process. Although the technique is a useful tool for modelling earthquake occurrence, the application of the Markov process has been limited. An example is the application of the continuous-time and continuous-state Markov model, in order to describe aftershock sequences as well as sequences of main events followed by aftershocks that are described by Vere-Jones (1966) and Knopoff (1971), respectively. Lomnitz-Adler (1983) used a simulation of Markov model to achieve a simplified representation of the spatial distribution of earthquakes on adjacent faults.

In this work, we employ a well-known stochastic process called the Markov chains. We will use the symbols p_{ij} and P to represent the transition probabilities from state to state and the matrix of the transition probabilities, respectively. Nevertheless, various quantities can be defined as states, which may correspond to a variety of energy releases levels, the magnitudes of the earthquakes, etc. In the present study, we define as states the seismic zones as described by Papadimitriou (1994).

The time period covered by the present study is 1900-1996. For these purposes, a catalogue of events was constructed taking into account the earthquakes listed in the catalogues of: a) Gutenberg and Richter (1954), b) Rothe (1969) and c) the ISC bulletins after the year 1964. This catalogue was then improved considering the magnitudes given by Pacheco and

Table 1. Schematic representation of the transition probabilities matrix. Frequency of visits in every cell and the transition probabilities between cells as derived from a Markov process. $n_{11},\ldots,n_{44}$ are the frequency of visits (which can be considered as earthquake occurrences) in every state, $p_{11},\ldots,p_{44}$ are the corresponding transition probabilities of visits from state *i* to state *j* and n_1, n_2, n_3, n_4 are the total number of visits in every row.

	state 1	state 2	state 3	state 4	row total
state 1	n_{11} p_{11}	n_{12} p_{12}	n_{13} p_{13}	n_{14} p_{14}	n_1
state 2	n_{21} p_{21}	n_{22} p_{22}	n_{23} p_{23}	n_{24} p_{24}	n_2
state 3	n_{31} p_{31}	n_{32} p_{32}	n_{33} p_{33}	n_{34} p_{34}	n_3
state 4	n_{41} p_{41}	n_{42} p_{42}	n_{43} p_{43}	n_{44} p_{44}	n_4

Sykes (1992). The present study is restricted to the shallow ($h \leq 60$ *Km*) events only.

The paper confines itself to the spatial and temporal earthquake occurrence in the area of southern Alaska and Aleutian Islands. It is aiming to establish the active or quiescence state, present state of zones on a quantitative basis through the computed transition probabilities.

A BRIEF REVIEW OF THE MARKOV PROCESS

Let p_{ij} be the probability that a Markov process that entered state *i* on its last transition will enter state *j* on its next transition as computed by the maximum likelihood estimators. The transition probabilities, which completely determine the Markov process, must satisfy the following properties:

$$p_{ij} \geq 0 \qquad i=1,2,3,\ldots,N,\ \ j=1,2,3,\ldots,N \tag{1}$$

and

$$\sum_{j=1}^{N} p_{ij} = 1 \tag{2}$$

where *N* is the total number of states in the system.

Whenever the process enters a state *i* the likelihood that it will go to state *j* at some future time is determined by the transition probability p_{ij} , which is estimated by:

$$p_{ij} = \frac{n_{ij}}{n_i} \tag{3}$$

where n_{ij} is the number of transitions from state *i* to state *j* and n_i is the number of transitions from state *i* to all the other states.

Thus the probability of being in some state *j* at a future time *t+s* is deduced from the knowledge of the state *i* at an earlier time *t* and is independent of the history of the process up to time *t* (Anagnos and Kiremidjian, 1988).

The frequency of visits in every state (seismic zone) and the transition probabilities between the states can be expressed through a matrix *P*. A general framework of a transition matrix (4X4) is presented in Table 1. According to the theory of Markov chains this is not a two-way matrix $\left(n_{ij} \neq n_{ji}\right)$ and one "reads" only from rows to columns. The number of visits *(n_{ij})* in the present work can be considered as earthquake occurrence (Anagnos and Kiremidjian, 1988). This is the significance of the method, because we can see for example how many times earthquakes migrated (visited), for example, from zone 2 to zone 3 or from zone 1 to 4, etc. It may be assumed that this represents a physical phenomenon and visits could be happened 3 or 4 or 16 times. We make it clear from the beginning in order to avoid any confusion that we

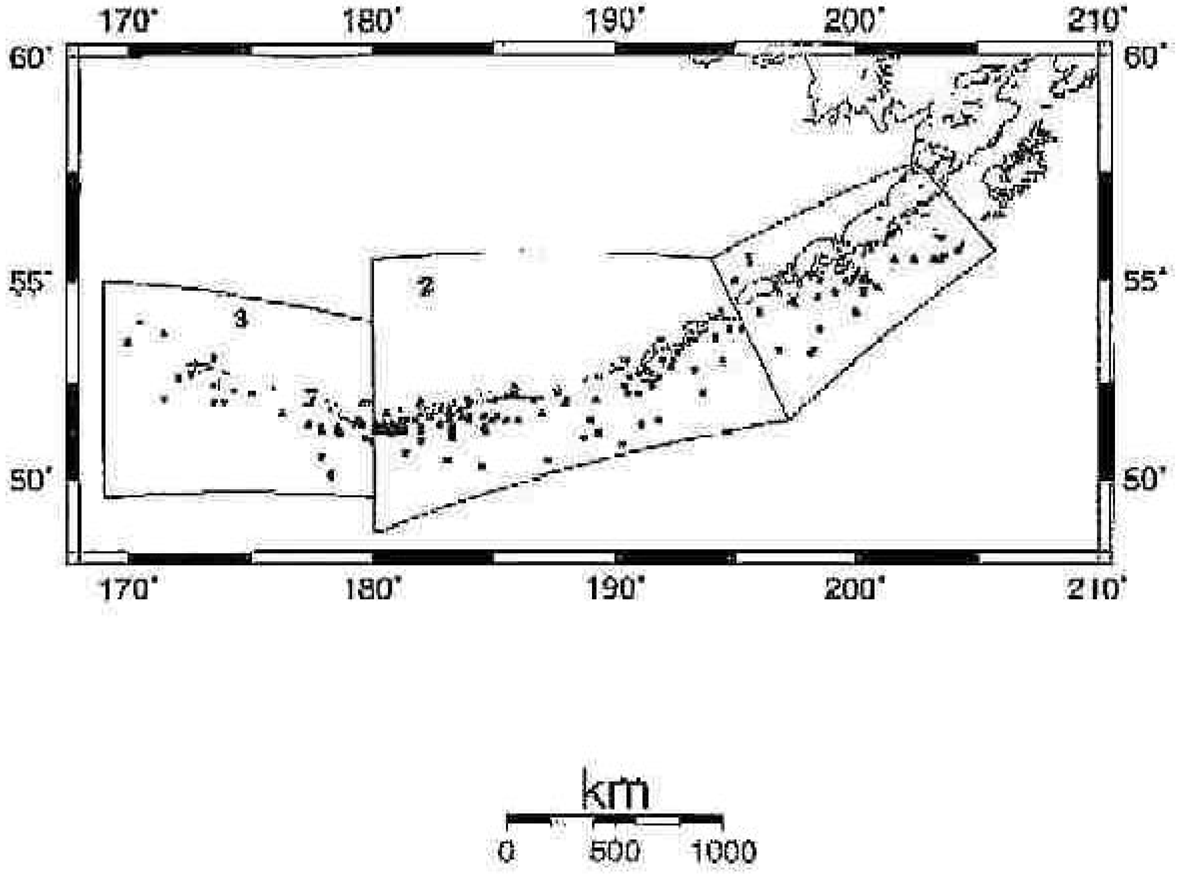

FIG. 1. The definition of the three examined zones. Epicentres of earthquakes with magnitudes M≥6.5 are depicted without any further details.

calculated the transition probabilities from 1 or 3 or 12 events.

A PATTERN OF SPATIAL AND TEMPORAL EARTHQUAKE OCCURRENCE: APPLICATION OF THE MODEL

The area of southern Alaska and Aleutian Islands is one of the most seismically active regions of the world. Tsapanos and Burton (1991) ranked the area of Alaska and the Aleutian Islands in the third position among 50 seismic regions of the world in terms of their seismic hazard. The area under investigation is bounded between the latitudes 48°-59° N and the longitudes 170°E-153°W and is divided in 3 zones (Fig. 1). Because of plotting package the longitudes appear in Figure (1) from 170° to 210°, which is equivalent with the real longitudes referred above. The spatial distribution of earthquakes in the whole area, the annotation number of each zone and the epicentre of the events with M≥8.0 along the year of occurrence are illustrated in Figure (2). The examined zones follow the ones given by Papadimitriou (1994). In accord to her division, we studied the zones 2, 3, and 4, which will be for our work zones 1, 2 and 3 respectively. The corresponding names of these zones are zone 1-Shumagin islands and Alaska Peninsula; zone 2-Andreanoff islands; and zone 3-Rat islands. Three very large earthquakes with M≥8.0 occurred in the area in the time span 1900-1996. These are the earthquakes of 1938 (in zone 1) and 1957 (in zone 2), both with magnitude M= 8.1, and the well-known event of 1965 (in zone 3) with M= 8.2.

If each one of these zones is considered to be a state in a Markov process, then the number of the zone in which it occurred could characterise every shock. Therefore we can examine aspects of the spatial and the temporal occurrence of the shocks quantitatively through the transition probabilities. We will verify if there is any existing pattern for strong (M≥6.5) earthquake occurrence in space and time. Earthquakes with M≥6.5 are taken into account, as such events cause damages and often casualties wherever they are generated. The frequency of visits (which can be considered as occurrence of earthquakes, as we referred above) in every zone and

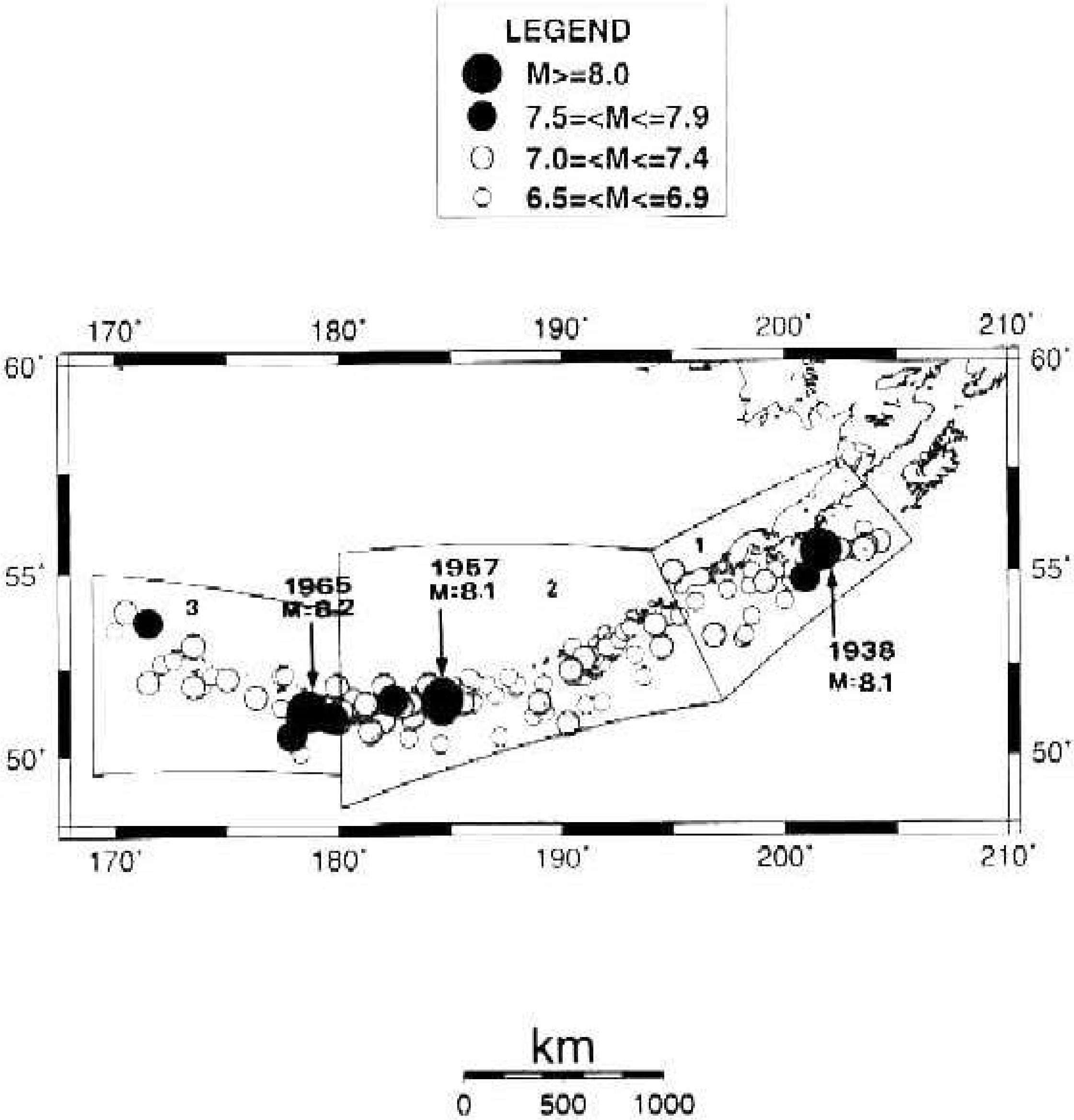

FIG. 2. The spatial distribution of earthquakes in southern Alaska and Aleutian Islands with magnitudes $M \geq 6.5$, which are used in the present study. The division of the examined area into three zones which are presented the three states, the epicentres of the very large earthquakes with $M \geq 8.0$ (large black circles) and the year of their occurrence are also depicted.

the transition probabilities (expressed as percentages) between the zones as calculated using a Markov process are presented in Table 2. The processes of three magnitudes cut-off [a) $M \geq 6.5$, b) $M \geq 6.7$ and c) $M \geq 7.0$] are tabulated in Table 2 for comparison purposes. By the inspection of Table 2a, we can observe that the earthquakes with $M \geq 6.5$ show a pattern of occurrence in space and time. This pattern illustrates that all the states (zones) show a preference visiting State 2. More clear observations are illustrated in Table 2b when we involved with earthquakes with $M \geq 6.7$. There are 7 visits (considered as earthquake occurrences) from zone 1 (state1) to zone 2, 12 visits from zone 2 to zone 3 and 6 visits from zone 3 to zone 1, with corresponding transition probabilities 41.2, 50.0 and 37.5. It is evident that an east-west (which from Alaska towards Aleutian Islands) pattern exists. The same pattern exists for earthquakes with $M \geq 7.0$ also, as it was demonstrated in the transition matrix in Table 2c. We remind that, we seek for a pattern of spatial and temporal distribution of large ($M \geq 7.0$) and very large ($M \geq 8.0$) earthquakes in the area. The number of shocks having magnitudes $M \geq 8.0$ is very limited for a Markov process. However, it can be assumed that the conclusions derived for earthquakes with $M \geq 7.0$ are valid for the largest events ($M \geq 8.0$) as

Table 2. Frequency of visits (earthquake occurrences) and the transition probabilities (in percent) in the examined zones of southern Alaska and the Aleutian Islands, for three magnitude thresholds: a) M≥6.5, b) M≥6.7 and c) M≥7.0.

a) for earthquakes with M≥6.5

	1	2	3	row total
1	6 31.6	8 42.1	5 26.3	19
2	6 17.1	15 42.9	14 40.0	35
3	7 29.2	11 45.8	6 25.0	24

b) for earthquakes with M≥6.7

	1	2	3	row total
1	6 35.3	7 41.2	4 23.5	17
2	2 8.3	10 41.7	12 50.0	24
3	6 37.5	5 31.3	5 31.3	16

c) for earthquakes with M≥7.0

	1	2	3	row total
1	2 28.6	4 57.1	1 14.3	7
2	1 7.7	5 38.5	7 53.8	13
3	4 40.0	3 30.0	3 30.0	10

well. Table 2c shows the existing east-west pattern with a very high probability (57.1%) for the transition from zone 1 to zone 2. This means that the process is in zone 1 where in 1938 the very large event (M=8.1) occurred and then it has "preferred" to visit zone 2 where in 1957 the next very large shock with the same magnitude generated. After that the earthquakes genesis the process moved to zone 3 with very high transition probability (53.8%) where the very large event with M=8.2 occurred in 1965. Combining the observations and the application of the Markov process the clear east-west migration pattern can be inferred for large (M≥7.0) and very large (M≥8.0) earthquakes. The pattern seems to prefer to return then to zone 1 with high transition probability 40.0%. The last high transition probability can be interpreted that zone 1 will be in the future the place of the next very large earthquake. This was supported with the fact that the Shumagin gap is still unbroken after the 1938 event according to Davies *et al*. (1981). The same authors estimated the repeat times for the rupture zone of the 1938 event to be 50 to 90 years. Based on the existence of the high seismic potential they suggested the occurrence of a large earthquake (M≥7.8) in the next one or two decades. Of course this suggestions implies that there is a low (but finite) probability that no large earthquakes will occur there for several decades. Also, Papadimitriou(1994) suggested that during the time period 1993-2003 the area will experience an earthquake of magnitude about M=8.2 with a high (60%) probability. The present study reinforces the classical paper of Kelleher(1970) who concluded that major earthquakes with M≥7.7 tend to progress in time from east to west.

THE TWO-STATE MARKOV MODEL IN THE THREE EXAMINED ZONES

It is well known that seismically active regions have periods of quiescence and activity. Here we present a model that each one of the three examined zones can be one of two states; these are defined as "inactive" and "active" and they will be represented as *0*-state and *1*-state, respectively. The Markov model has the advantage to allow derivation of the transition probabilities from "active' to "inactive" periods and vice versa. It can also represent the current state of each zone. There are four possible conditions: a) visits from *0* to *0* (with transition probability p_{oo}) which means transition from inactive to inactive period, b) steps from *0* to *1* (p_{o1}) and *1* to *0* (p_{1o}) which is transitions from inactive to active and from active to inactive periods, respectively, and c) transition from *1* to *1* (p_{11}) is that of an active period followed by an other active one. The general form of a transition matrix, *P*, of a two-state model is:

P=

	0	1	
0	p_{oo}	p_{o1}	n_1
1	p_{1o}	p_{11}	n_2

Let *W* be the random variable which represents the number of successive active years:

$$\begin{aligned}\text{Prob}\{W=n\}&=\text{Prob}\{X_2=...=X_n=1,\ X_{n+1}=0\\ &|X_1=1\}=\text{Prob}\{X_2=1\ |\ X_1=1\}\cdot\text{Prob}\{X_3=1\\ &|X_2=1\}\ \cdot ...\\ &\text{Prob}\{X_n=1|\ X_{n-1}=1\}\cdot\text{Prob}\{X_{n+1}=0\ \ |\ X_n=1\}=\\ &p_{11}^{n-1}\cdot p_{1o}\end{aligned}$$

that is, the distribution of the random variable *W* is a geometric one with probability of success p_{11} and consequently its mean is:

$$E(W)=\sum_{n=1}^{\infty} n p_{11}^{n-1} p_{1o} \tag{4}$$

where

$$\frac{1}{1-p_{11}}=\frac{1}{p_{1o}},$$

given that

$$p_{1o}+p_{11}=1.$$

We can therefore calculate the transition matrices of the three zones in which southern Alaska and Aleutian Islands are divided. The visits between the active and inactive states are presented through transition probabilities. Earthquakes with M≥6.7 are processed:

a) ZONE 1-Shumagin Islands and Alaska Peninsula

	0	1	
0	46 78.0	13 22.0	59
1	13 86.7	2 13.3	15

That the mean duration of an active period is:

$$\frac{1}{p_{1o}}=\frac{1}{0.867}=1.2\ \text{yrs.}$$

a) ZONE 1-Shumagin Islands and Alaska Peninsula

	0	1	
0	46 78.0	13 22.0	59
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That the mean duration of an active period is:

$$\frac{1}{p_{1o}} = \frac{1}{0.867} = 1.2 \text{ yrs.}$$

b) ZONE 2- Andreanof Islands

	0	1	
0	50 74.6	17 25.4	67 22
1	17 60.7	6 39.3	28

where the mean active duration is:

$$\frac{1}{p_{1o}} - \frac{1}{0.607} = 1.6 \text{ yrs.}$$

c) ZONE 3- Rat Islands

	0	1	
0	55 78.6	15 21.4	70
1	15 71.4	6 28.6	21

with a mean active duration equal to:

$$\frac{1}{p_{1o}} = \frac{1}{0.714} = 1.4 \text{ yrs.}$$

The transition probabilities in the above matrices are expressed as percentages, and they are very useful for seismic hazard assessment. Considering the current state of seismic activity we can conclude whether the next period is active or inactive in a quantitative way through the transition probabilities.

We can suggest that inactive periods dominate the three examined zones. The first zone is in inactive state given that an earthquake with M=7.0 in 1964 occurred there and the probability for the transition to an active state is 22%. The last activity in the second zone is relatively recent, given that the zone experienced a shock with M=7.6 in 1996. The visit to an active state in this zone has a transition probability of 25%. Zone 3 is still inactive since 1981 where an event with M=7.0 occurred and we calculated that the transition probability for the zone to enter the active state again is 21.4%.

DISCUSSION AND CONCLUSIONS

The Markov model is applied in the present study in order to search the distribution of large ($M \geq 7.0$) earthquakes in space and time for the seismic area of southern Alaska and Aleutian Islands for the time period 1900-1996.

A pattern, which suggests that the distribution of earthquakes in space and time tends to progress in an east-west direction, is defined, reinforcing earlier studies. For the purposes of this study, the area under investigation was divided into three zones that are defined as states. The visits from one state to another clearly revealed the pattern (Fig. 3). The computed transition probabilities illustrated this pattern in a quantitative way. Sykes(1971) observed that the rupture zones of five events appeared to form a space-time sequence that progressed from 155^{o}W in 1938 to 171^{o}E in 1965. Also he suggested that these rupture zones are more than 250 km long and they tend to abut each other without much overlap. Another characteristic feature is that great earthquakes occur as a sequence of shocks in a relatively short period of time followed by a period without great earthquakes until the next sequence begins (Sykes *et al*., 1980). A model for this east-west migration of large and very large earthquakes is given by Li and Kisslinger(1985) who studied the 1938-1965 series of large earthquakes occurrence in the examined area. According to this model, stress and strain were building up in the area due to tectonic plate movement. The first interruption of this quiet period occurred in 1938 when a very large earthquake ruptured a 300-km segment east of the Shumagin gap. Stress may have transferred by this shock to its adjacent segments causing the events of 1946 and 1948 that followed by two very large earthquakes in 1957 and 1965. The westward migration of these shocks may be regarded as a result of delay triggering through stress diffusion. The same authors concluded that zones 1 and 2 have the highest expectation of seismic rupturing in the near future.

The pattern tends to return back to zone 1 with high (40%) transition probability. This fact indicates the site of the next large event. The site (zone 1) of the

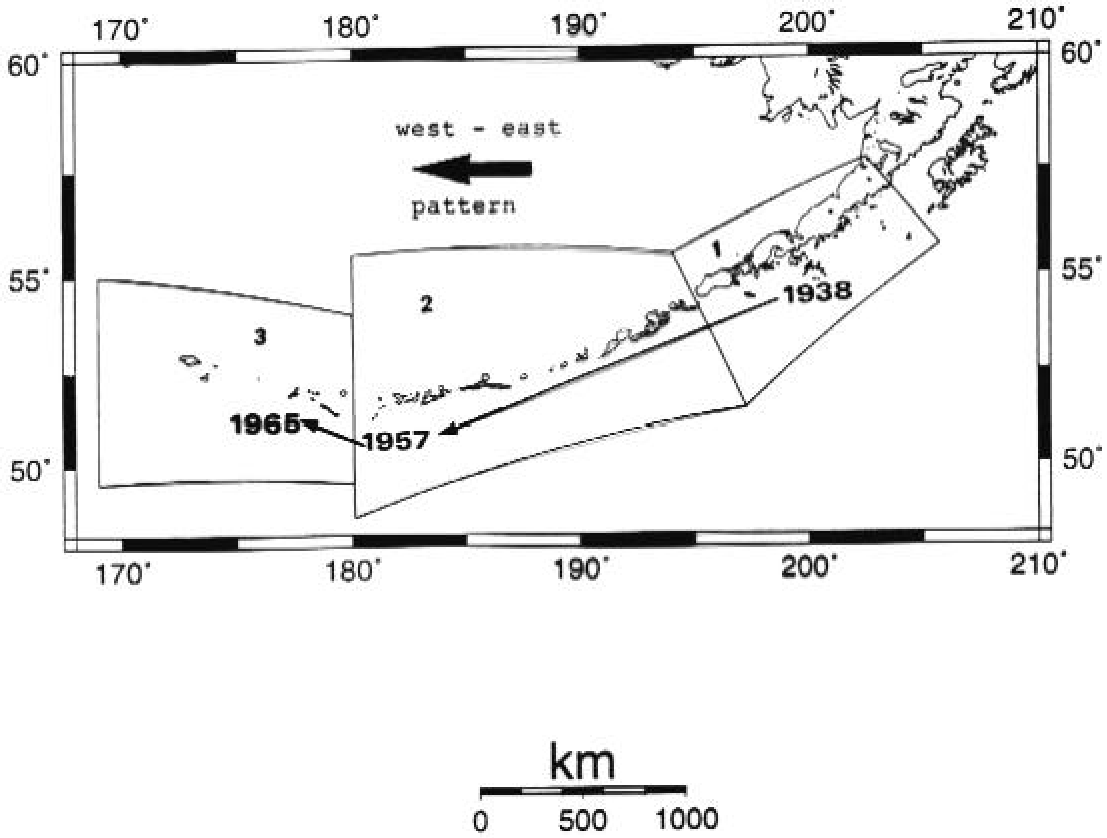

FIG. 3. Sketch of the east-west pattern found for the genesis of large earthquakes in the examined area. Solid arrows illustrate the visits from zone to zone (which is from state to state), while the pattern of the east-west migration of the occurrence of the earthquakes is indicated with a large bold arrow.

next large event was also pointed by Sykes *et. al.* (1980) and Davies et al. (1981) who estimated that a large event would occur in the next 10-20 years. Nishenko(1991) found that the conditional probability of a large earthquake recurrence in this zone is 41% within the next 20 years (1989-2009).

A two state Markov model was further applied to the data. The model describes the active and inactive periods in each of the three zones. The two-state model can be viewed as a switch which in its "on" position indicates an active period, while its 'off" position shows the inactive periods. The mean duration of the active periods in each one of the three zones is estimated. Various combinations between these two states can be made in order to define visits from inactive to active periods and vice versa. All the zones are currently in inactive state, but with relatively fair probabilities for a transition to active periods. The inactive-active transition probabilities for the three zones are: 22% (zone 1), 25% (zone 2) and 21% (zone 3). Papadimitriou(1994) has established the probabilities of the expected earthquakes with $M \geq 7.5$ during the decade 1993-2002. According to this author: an earthquake with M=8.2 will occur with probability 60% in zone 1, an earthquake of M=8.1 will strike zone 2 with probability 25%, while a shock with M=7.7 will be generated with probability 48% in zone 3.

The transition probabilities computed in the present paper make this model useful in the seismic hazard assessment studies.

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