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Interpretation of magnetic anomalies from dipping dike model using inverse solution, power spectrum and Hilbert transform methods

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Abstract: *A theoretical and a field study were performed for the interpretation of the vertical component magnetic anomalies from dipping dike model with infinite depth extent. Damped least squares inverse solution, Hilbert transform and power spectrum methods were used in interpretation. The parameters affecting the power spectrum of the analytical model response were determined and discussed. Damped least squares inverse solution, Hilbert transform and power spectrum methods were examined on a theoretical anomaly of a magnetic dike model. The methods were then applied to a magnetic field anomaly from northeastern Turkey to obtain the dike parameters. Only the depth of the dike can be obtained in the power spectrum method, but it is also possible to find out the width of the structure if the slope of the dike is known. In the inversion method, it is possible to obtain all the dike parameters. The depth, width and the location of the origin of the dike can be obtained by Hilbert transform method. It was observed that the results obtained from all of the three methods are evidently consistent with each other.*

Key Words: *Magnetic dike model, Parameter solutions, Least squares inverse solution, Hilbert Transform, Power spectrum method.*

INTRODUCTION

The dike model is one of the most commonly used models in the interpretation of magnetic anomalies. In inverse modeling, a geometrical model is chosen with initial estimates of the body parameters, and then the process is iteratively advanced until a satisfactory fit is obtained between observed and calculated anomalies. Several methods, such as the gradient method, ridge regression, Gauss method and singular value decomposition have been used to determine the body parameters automatically. Rao et al. (1973) used the gradient method to determine the change in parameter vector required to account for the residuals. Johnson (1969) used damped least squares ridge regression to optimize magnetic anomalies caused by 2D bodies of polygonal cross section. Won (1981) used the Gauss method to minimize the residuals for anomalies caused by dike like bodies. Khurana et al. (1981) studied the application of the Marquardt (1963) algorithm in the inversion of magnetic anomalies due to dike models in the frequency domain. In this study, the body parameters were determined by using the inversion method given by Marquardt (1963).

The Hilbert transform method has several applications in potential field methods. Nabighian (1972) focused on the determination of the vertical magnetic field from its horizontal component, or vice versa, using Hilbert transform in magnetic method. Stanley and Green (1976) and Stanley (1977) proposed an interpretation procedure based on the vertical and horizontal gradients. Then Sundararajan et al. (1985) used Hilbert transform in the interpretation of magnetic anomalies of various components. Ofoegbu and Mohan (1990) performed 3D Hilbert transform in the interpretation of aeromagnetic anomalies. The interpretation procedure given by Mohan et al. (1982) was used in this study.

The application of the power spectrum method to potential field data was proposed by Bhattacharyya (1966) and Spector and Bhattacharyya (1966). The determination of the anomalous body depth was then given by Spector and Grant (1970).

In this study, interpretation of magnetic anomalies arising from dipping dike model is studied by using damped least squares inverse solution, Hilbert transform and power spectrum methods. These methods are examined and compared on a theoretical anomaly of a magnetic dike model, and then a field example from the northeast of Turkey is also included.

THE METHODS EMPLOYED

The mathematical expression for a vertical magnetic field anomaly V(x) due to a uniformly magnetized dipping dike of infinite depth extent (Fig. 1) is given by (Grant and West, 1965)

$$V(x) = 2kF_e \sin\beta \left[\cos I \left(\tan^{-1}\frac{x+b}{h} - \tan^{-1}\frac{x-b}{h} \right) + \frac{1}{2}\sin I \left(\ln \frac{h^2+(x+b)^2}{h^2+(x-b)^2} \right) \right] \quad (1)$$

where F_e is the vertical component of the local ambient field, k is the susceptibility contrast, I is the inclination of the Earth 's magnetic field and β is the dip of the dike as in Figure 1.

Damped Least Squares Inversion Method

The inverse problem in geophysics involves the selection of a geometrical model for which a mathematical formula can be derived to calculate the model response using the initial values of body parameters. The optimization procedure changes the initial body parameters iteratively to reach a model, which fits the

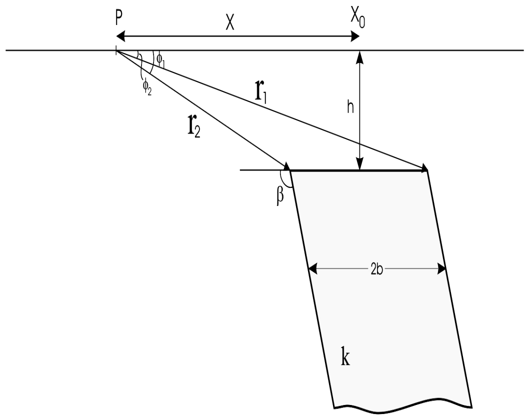

FIG. 1. Schematic illustration of the geometry of the dike model. Parameters estimated are burial depth (h), dip angle (β), surface projection mid-point (X_0) of upper surface, half-width (b) and susceptibility (k) of the dike.

observed data (Marobhe, 1989). In nonlinear inverse problems of geophysics, the observed values are considered as,

$$f = f(X, \beta) + \varepsilon \quad (2)$$

where X is the variable vector, β is the model parameter vector and ε is the error vector. Using least squares inverse solution, the nonlinear approximation can be performed as

$$G = \delta A \quad (3)$$

where $A = P^T P$ and $G = P^T(f - \hat{f})$, P is the Jacobian matrix, f and $\hat{f}$ are the observed and estimated values, respectively. Marquardt (1963) solved equation (3) in order to find out δ error vector by expressing equation (3) as

$$G = (A + \lambda I)\delta \quad (4)$$

where I is unit matrix and λ is the damping factor.

Hilbert Transform Method

The Hilbert transform of the vertical magnetic effect of a dike like body with infinite depth extent is given by (Thomas, 1969),

$$H(x) = \frac{1}{p}\int_0^{\infty}[\mathrm{Im}F(w)\cos wx - \mathrm{Re}F(w)\sin wx]\,dw \quad (5)$$

where F(ω) is the Fourier transform of the vertical magnetic effect V(x), defined as

$$F(\omega) = \int_{-\infty}^{\infty} V(x)e^{-i\omega x}dx = \mathrm{Re}F(\omega) + i\,\mathrm{Im}F(\omega) \quad (6)$$

Substituting the equation (1) into the equation (6), the real and the imaginary parts of the Fourier transform of the vertical magnetic effect of the dike are obtained as

$$\mathrm{Re}\,F(\omega) = \frac{4\pi kF_e}{\omega}\sin(\hat{a})\cos I \sin(\omega b)e^{-\omega h} \quad (7a)$$

$$\mathrm{Im}\,F(\omega) = \frac{4\pi kF_e}{\omega}\sin(\beta)\sin I \sin(\omega b)e^{-\omega h} \quad (7b)$$

Substituting equations (7a) and (7b) into equation (5), the Hilbert transform is obtained as (Mohan et al., 1982)

$$H(x) = 2kF_e\sin(b)\left[\sin I\left(\tan^{-1}\frac{x+b}{h} - \tan^{-1}\frac{x-b}{h}\right) + \frac{1}{2}\cos I\left(\ln\frac{h^2+(x+b)^2}{h^2+(x-b)^2}\right)\right] \quad (8)$$

In interpretation, the derivative of the analytical expression of vertical field magnetic anomaly of equation (1) is given by

$$V'(x) = \frac{dV}{dx} = 2kF_e\sin(b)\left[\cos I\left(\frac{h}{h^2+(x+b)^2} - \frac{h}{h^2+(x-b)^2}\right) + \sin I\left(\frac{(x+b)}{h^2+(x+b)^2} - \frac{(x-b)}{h^2+(x-b)^2}\right)\right] \quad (9a)$$

and the derivative of the Hilbert transform of equation (8) is given by

$$H'(x) = \frac{dH}{dx} = 2kF_e\sin(\beta)\left[\sin I\left(\frac{h}{h^2+(x+b)^2} - \frac{h}{h^2+(x-b)^2}\right) + \cos I\left(\frac{(x+b)}{h^2+(x+b)^2} - \frac{(x-b)}{h^2+(x-b)^2}\right)\right] \quad (9b)$$

In this study, we used the intersection points x_1 and x_2 from V'(x) and H'(x) respectively. Using x_1 and x_2, depth to the top (h) and the half-width of the dike (b) are obtained as (Mohan et al., 1982),

$$h = \frac{x_1 + x_2}{2} \quad (10)$$

$$b = \sqrt{\frac{(x_1 - x_2) - (x_1 - x_2)^2}{4}} \quad (11)$$

In addition, the location of the origin of the anomaly (X_0) is necessary for the purpose of interpretation. The origin location can be determined from maximum point of the derivative curve of the Hilbert transform.

The Power Spectrum Method

Previous studies on power spectrum method showed that the depth of the structure could be determined by using slope of the power spectrum curve of the field anomalies. In all kinds of potential field methods, the relationship between the slope of the power spectrum curve and the depth of the anomalous body (h) is given as slope=-2h. In this study, the parameters affecting the power spectrum of the magnetic anomalies from a dipping dike were obtained. In general form, the power spectrum can be defined as

$$E(\omega) = \mathrm{Re}\,F(\omega)^2 + \mathrm{Im}\,F(\omega)^2 \quad (12)$$

Using the equations (7a) and (7b) we can obtain the analytical formula of the power spectrum of a magnetic dike model by

$$E(w) = \left(\frac{4\eth kF_e}{w}\right)^2 \sin^2(wb)\,e^{-2wh}\sin^2(b)\cdot(\cos^2(I) + \sin^2(I)) \quad (13)$$

Substituting $\cos^2(I) + \sin^2(I) = 1$ and taking the logarithm of (13) for linearising,

$$\ln[E(w)] = \ln\left[\left(\frac{4\pi k F_e}{w}\right)^2\right] + \ln[\sin^2(wb)] - 2wh + \ln[\sin^2(b)] \quad (14)$$

In equation (14), the first and the last terms are constant; therefore, we can examine the effects of remnants to the power spectrum. The results are shown in Table 1 for ω=0.1 and ω=0.2, h=100 m, F_e=40400 γ, k=0.01 cgs, β=80° and b=75 m.

It is clear from Table 1 that the term, which most strongly affects the slope of the power spectrum curve, is 2ωh. Therefore, one can estimate the depth of the dike h, using the slope of the power spectrum curve from the formula slope=–2h. Furthermore, if we can estimate the slope of the dike β, than we can find the half-width of the dike b, using equation (14).

Table 1: Examination of the parameters affecting the power spectrum of analytical formula of the dike. ω is the angular frequency, b is the half-width and h is the depth of the dike.

ω	ln sin^2 (ωb)	2ωh
0.1	0.017	20
0.2	0.067	40

THEORETICAL EXAMPLE

In this study, the advantages of the three methods were examined for the interpretation of magnetic anomalies from dike like model. For this purpose, the model parameters were obtained and compared on a theoretical vertical component magnetic anomaly from a dipping dike model for all the three methods. Therefore, a magnetic anomaly was computed for h=100 m, F_e=40400 γ, X_0=500 m, k=0.01 cgs, β=80°, I=55° and b=75 m and station interval dx=10 m (Fig. 2). The horizontal derivatives of this anomaly curve and its Hilbert transform is shown in Figure 3, and the power spectrum curve of the anomaly is shown in Figure 4. The model parameters obtained from inversion, Hilbert transform and power spectrum methods are shown in Table 2.

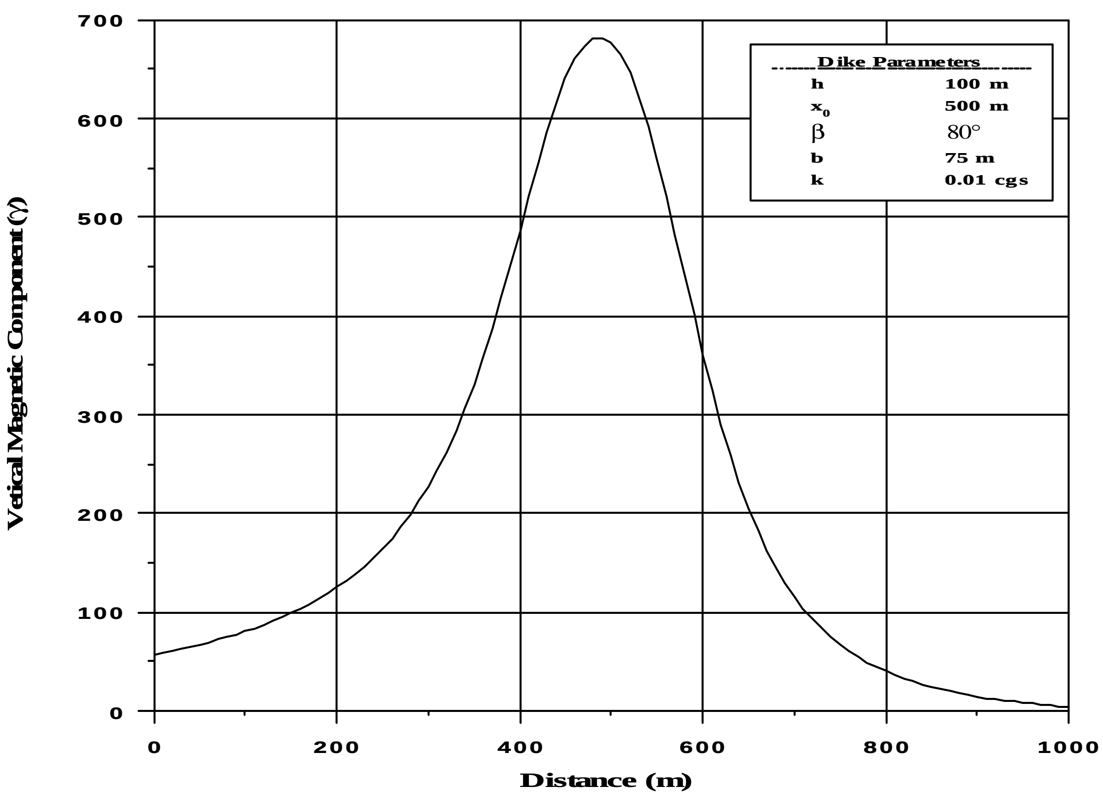

FIG. 2. Theoretical vertical component magnetic anomaly arising from dipping dike model. The body parameters used to compute the anomaly are also indicated.

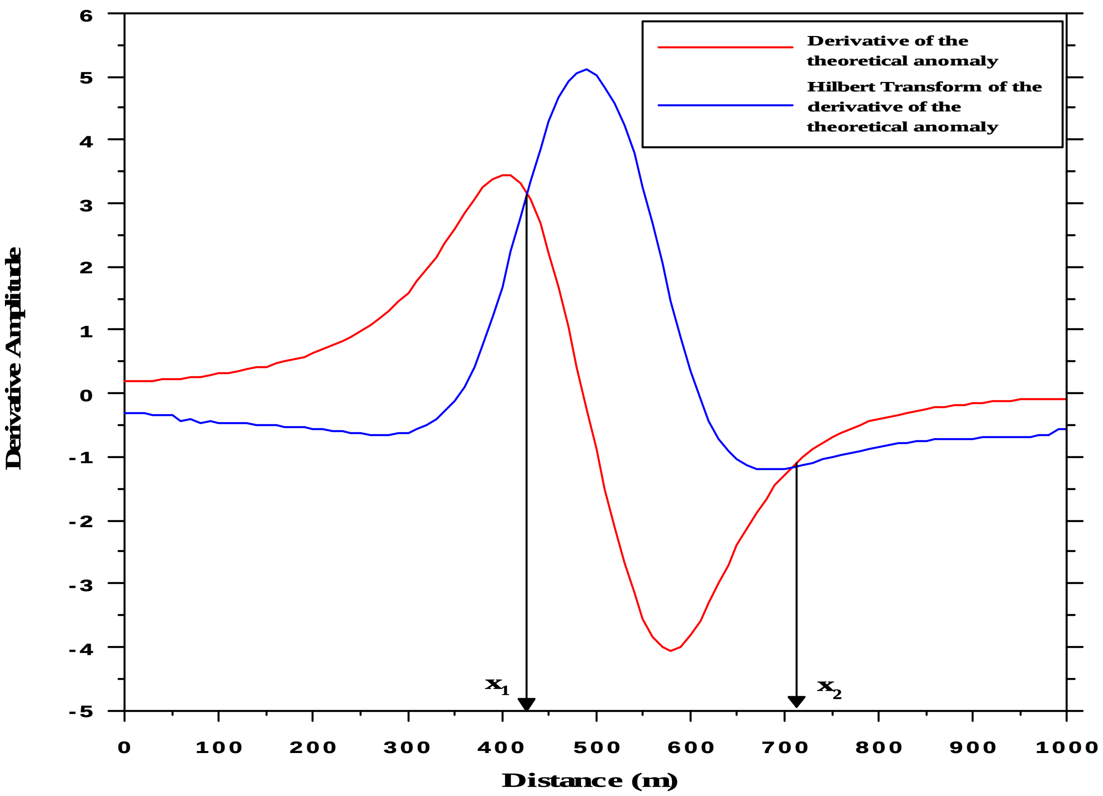

FIG. 3. Derivative of the theoretical magnetic anomaly in Figure 2 and its Hilbert Transform.

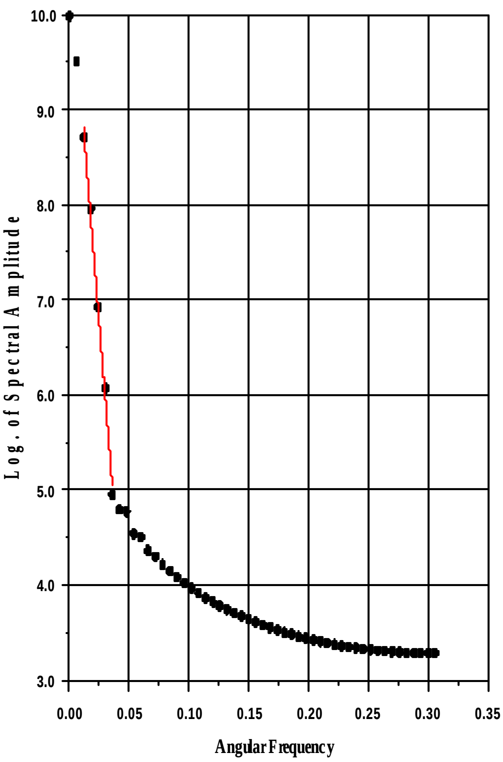

FIG. 4. The power spectrum curve of the theoretical magnetic anomaly in Figure 2. The red line corresponds to the slope of the power spectrum curve, which is used to estimate the depth of the dike.

Table 2: Comparison of the parameters determined by inverse solution, Hilbert Transform and power spectrum methods for theoretical example.

	h	b	β	X_0	k
Inverse Solution	100 (m)	75 (m)	80°	500 (m)	0.01
Hilbert Transform	90 (m)	80 (m)		490 (m)	
Power Spectrum	100 (m)	70 (m)			

FIELD EXAMPLE

A vertical component magnetic anomaly of Bayburt-Sarýhan (Northeastern Turkey) skarn zone was considered in order to determine the dike parameters using damped least squares inverse solution, Hilbert transform and power spectrum methods.

Geology of the region

In the study area, a series of granodiorite, limestone, volcanic sediments and tuffs are present as main geological units (Keskin et al., 1989; Arslan, 1994). Early Lias granodiorite unit exhibiting evident relief causes a very steep topography in the region. The limestone is Cretaceous in age and shows a fractured structure. Volcanic sediments form a border with limestone in the eastern part of the region in North to South direction. In the southwestern part of the region, magnetite mineralization was observed in a skarn zone between the limestone and the granodiorite unit (Fig. 5a).

Interpretation of the Field Anomaly

The body parameters from the vertical component magnetic anomaly of Bayburt-Sarýhan (northeast of Turkey) skarn zone were determined. The results were compared with those of Aydýn and Geliþli (1996) who performed a study in the same region using a total field magnetic survey.

Because the mineralization developed along the skarn zone, granodiorites at the western part of the zone and the volcanic sediments at the eastern part of the zone are present. Aydýn and Geliþli (1996) performed some anisotropic magnetic susceptibility measurements in the region and showed that the granodiorite unit had a magnetic susceptibility 6 times greater than that of volcanic sediments ($3200x10^{-6}$ and $575x10^{-6}$ cgs, respectively). The vertical component magnetic anomaly map is shown in Figure 5b. The A-A' profile from field anomaly map of Bayburt-Sarýhan skarn zone is shown in Figure 6. Two intersection points of the horizontal derivatives of the anomaly and its Hilbert transform are shown in Figure 7. The intersection points are found to be x_1=260 m and x_2=-60 m. The power spectrum curve of the field anomaly is shown in Figure 8. The field and calculated anomalies, obtained from damped least squares inverse solution, are shown in Figure 9. There is an evident fit between observed and computed anomalies. The body parameters obtained from all three methods used are given in Table 3. As seen in Table 3, the obtained underground parameters show an eastward dipping dike with a slope of about 110°, depth of about h=100 m and a half-width of about b=75 m. The susceptibility of the dike is quite high (k=0.01 cgs approximately) as expected for a magnetite mineralization. These parameters are also consistent with that of Aydýn and Geliþli (1996) and geological observations of the study area.

Table 3: Comparison of the parameters determined by the methods used in this study and those from Aydýn and Geliþli (1996) for the field anomaly.

	h	b	β	X_0	k
Inverse Solution	97 m	76 m	111°	397 m	0.009 cgs
Hilbert Transform	100 m	75 m		400 m	
Power Spectrum	98 m	70 m			
Aydýn and Geliºli (1996)	94 m	66 m	130°	405 m	0.0105 cgs

CONCLUSIONS

In power spectrum method, only the depth of the dike can be obtained, but it is also possible to find out the width of the structure when the slope of the dike is known. The damped least squares inversion method makes it possible to obtain whole dike parameters simultaneously. Therefore, it provides more information on the parameters of the dike model with respect to the Hilbert Transform and power spectrum methods. However, the initial parameters of the inversion should be chosen appropriately in order to make the inversion convergent. In the Hilbert transform method, the values obtained from intersection points of the Hilbert transform and the derivative of the anomaly curves were used in the equations from analytical solution of the Hilbert Transform of the model response.

The methods included in this study are based on the parameter estimation of the solutions of analytical formula of the dike problem in magnetics. The result parameters from each method are consistent. Because the solutions of the potential field data are non-unique,it could

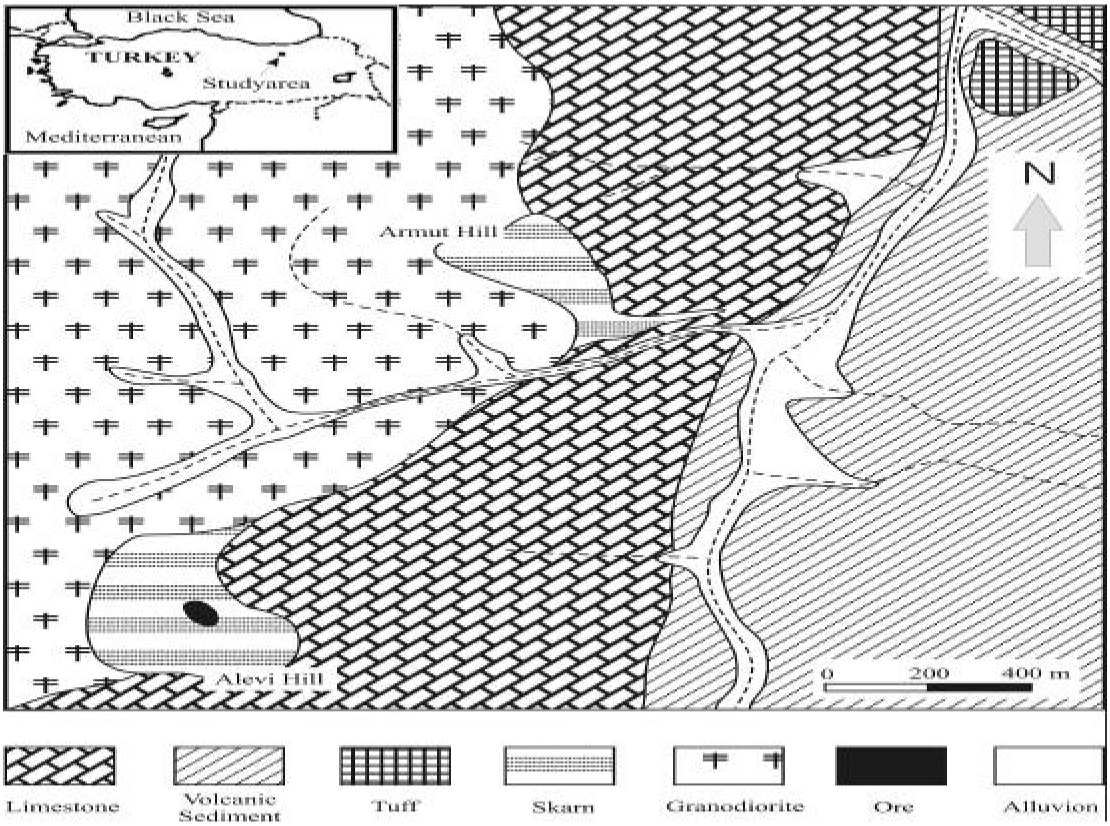

FIG. 5(a): Geological map of the study area showing the skarn-type magnetit mineralization between granodiorite and limestone units

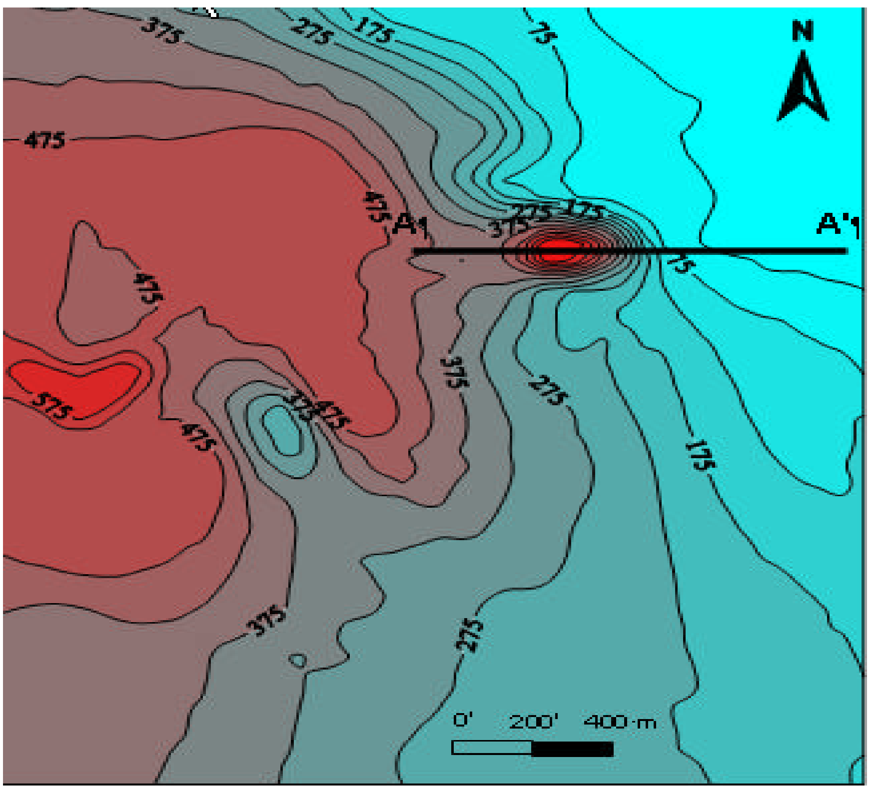

FIG.5(b): Vertical component magnetic map of the study area and the location of A-A' profile. Contour interval is 50 γ.

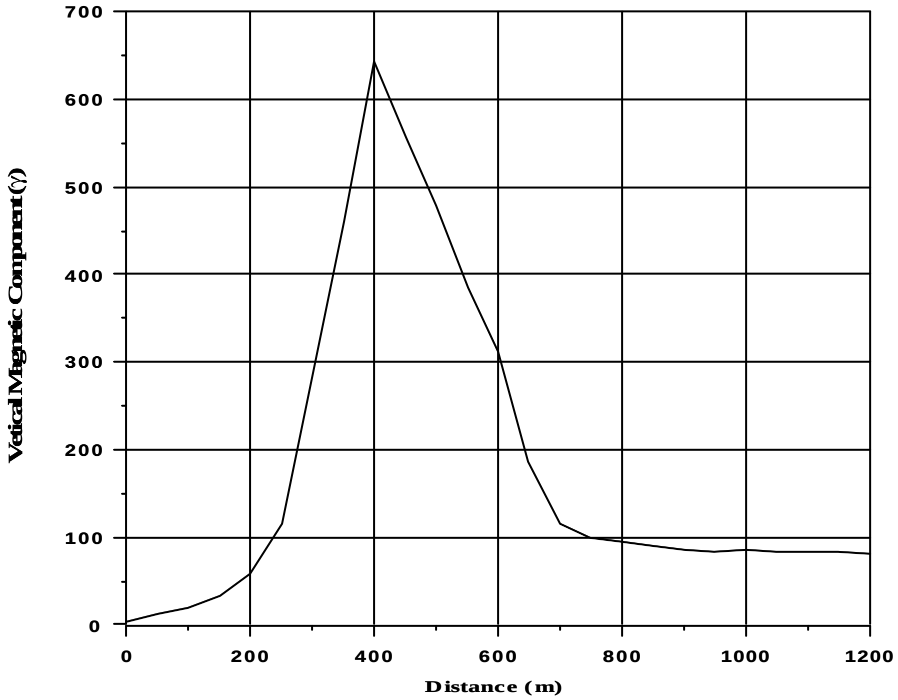

FIG. 6: Magnetic field anomaly of Bayburt-Turkey Skarn mineralization along the A-A' profile in Figure. 5.

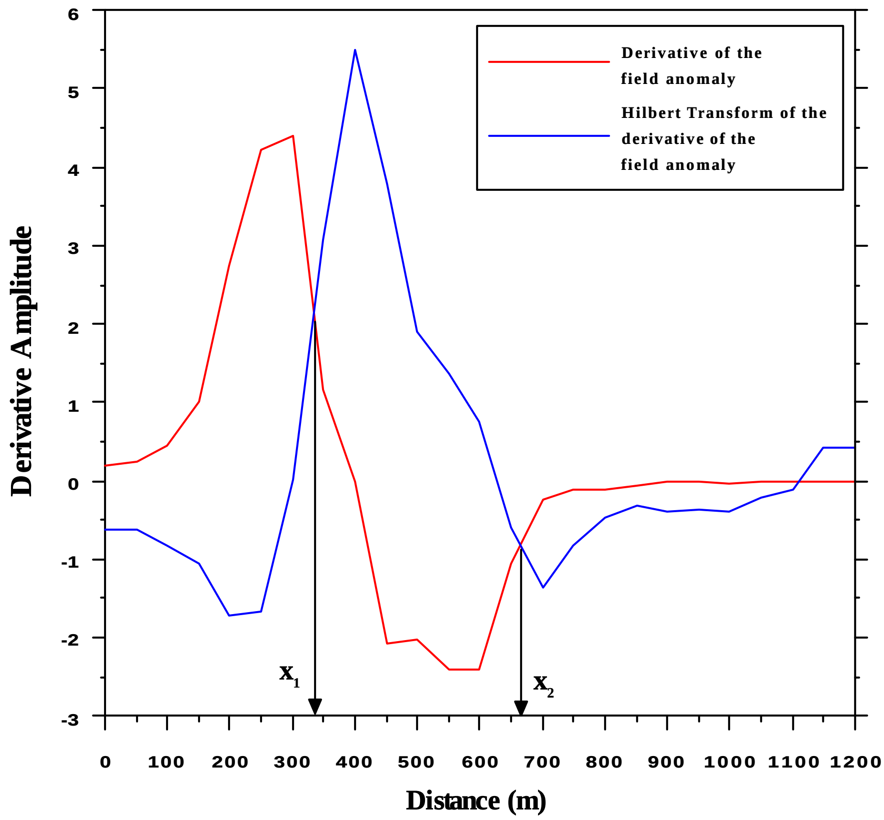

FIG. 7: Derivative of the field magnetic anomaly along the A-A' profile in Figure 6 and its Hilbert Transform.

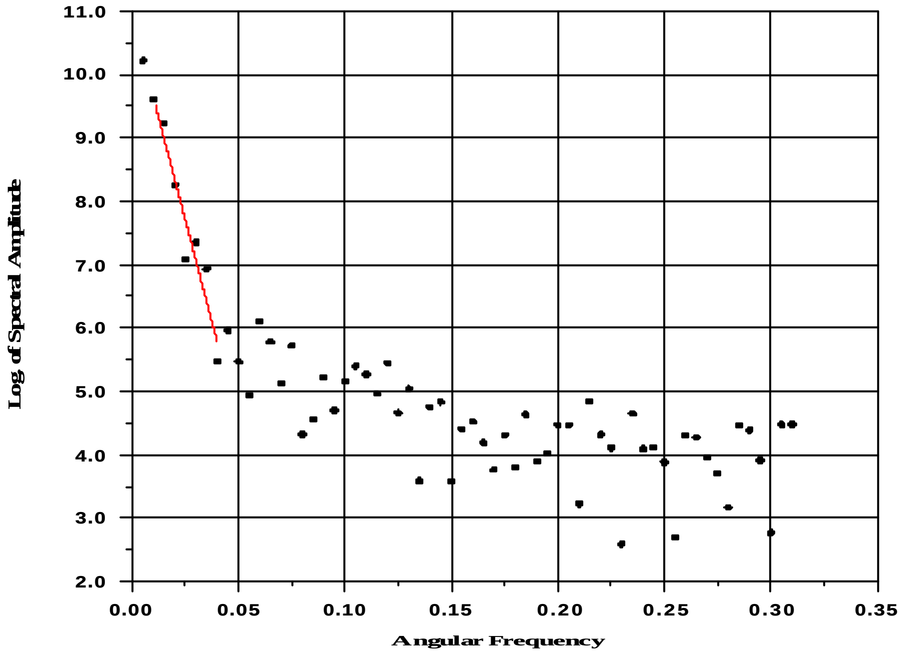

FIG. 8: The power spectrum curve of the field magnetic anomaly in Figure 6. The depth of the dike is estimated using the slope of the red line.

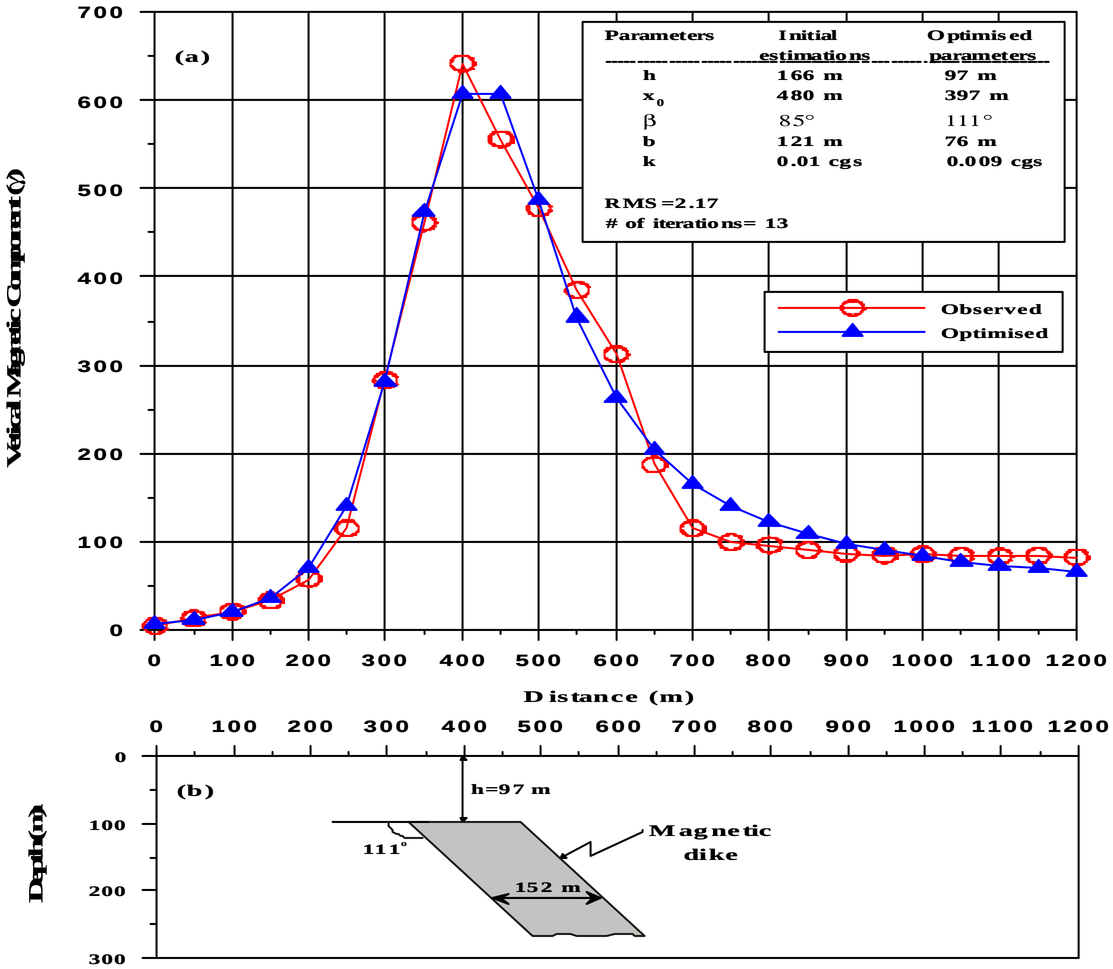

FIG. 9: (a) Inversion results of the field anomaly together with the initial and optimized parameters and **(b)** Geological cross section showing the parameters obtained.

be concluded that proceeding of more than one parameter solution method for potential field data is feasible and can supply more suitable and correct underground parameters.

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REFERENCES

Arslan, Z., 1994, Geology of Bayburt and Surrounding Areas and Granites, PhD Thesis, Karadeniz Technical University, Trabzon (in Turkish).

Aydýn, A. and Geliþli, K., 1996, Magnetic studies for the skarn zone of Sarýhan-Bayburt: Jeofizik, **10**, 41-51 (in Turkish with English Abstract).

Bhattacharyya, B.K., 1966, Continuous spectrum of the total magnetic field anomaly due to a rectangular pirismatic body: Geophysics, **31**, 97-121.

Johnson, W.W., 1969, A least squares method of interpreting magnetic anomalies caused by two dimensional structures: Geophysics, **34**, 65-74.

Keskin, I., Korkmaz, S., Gedik, I., Ateº., M., Gök, L., Küçükmen, M. and Erkal, T., 1989, Geology of Bayburt and Surrounding Areas, MTA, Geol. Prosp. Div., Ankara (in Turkish).

Khurana, K.K., Rao, S.V.S and Pal, P.C., 1981, Frequency domain least squares inversion of thick dike magnetic anomalies using Marquardt algorithm: Geophysics, **46**, 1745-1748.

Marobhe, I.M., 1989, Optimisation of Magnetic Anomalies via Singular Value Decomposition, in Geophysical Data Inversion Methods and Applications, ed. A. Bvogel, Bertelsmann Pub. Co., Germany.

Marquardt, D.W., 1963, An algorithm for least squares estimation of nonlinear parameters: J. Soc. Indust. Appl. Math., **11**, 431-441.

Mohan, N.L., Sundararajan, N. and Rao, S.V.S., 1982, Interpretation of some two-dimensional magnetic bodies using Hilbert Transform: Geophysics, **47**, 376-387.

Nabighian, M.N., 1972, The analytic signal of two-dimensional magnetic bodies with polygonal cross-section, its properties and use for automated anomaly interpretation: Geophysics, **37**, 507-512.

Ofoegbu, C.O. and Mohan, N.L., 1990, Interpretation of aeromagnetic anomalies over part of Southeastern Nigeria using three-dimensional Hilbert Transformation: Pure and Applied Geophysics, **134**, 13-29.

Rao, B.S.R., Murthy, I.V.R and Rao, C.V., 1973, Two methods for computer interpretation of magnetic anomalies of dikes: Geophysics, **38**, 710-718.

Spector, A. and Bhattacharyya, B.K., 1966, Energy spectrum and autocorrelation function of anomalies due to simple magnetic models: Geophy. Prosp., **14**, 242-272.

Spector, A. and Grant, F.S., 1970, Statistical models for interpreting aeromagnetic data: Geophysics, **35**, 293-302.

Stanley, J.M. and Green, R., 1976, Gravity gradients and the interpretation of Truncated Plates: Geophysics, **41**, 1276.

Stanley, J.M., 1977, Simplified gravity interpretation by gradients the geological contact: Geophysics, **42**, 1230-1235.

Sundararajan, N., Mohan, N.L., Raghava, M.S.V., and Rao, S.V.S., 1985, Hilbert Transform in the interpretation of magnetic anomalies of various components due to a thin infinite dike: Pure and Applied Geophysics, **123**, 557-566.

Telford, W.M., Geldart, L.P., Sheriff, R.E. and Keys, D.A., 1992, Applied Geophysics, Cambridge Univ. Press.

Thomas, J.B., 1969, An Introduction to Statistical Communication Theory, New York, John Willey and Sons Inc.

Won, I.J., 1981, Application of Gauss's method to interpretation of magnetic anomalies of dipping dikes: Geophysics, **46**, 211-215.